Date: Tuesday, February 26, 2019

Chapter 5

Example: Showcase Showdown

Bob Barker

Aim: Win both prizes
↳ Bid within $250 below the true price of your prizes.

Data: We observe the prize of similar prices from the last show.

Total: True price $\sim N(\mu_p, \sigma_p)$; assume $\mu_p = 35{,}000$
$\sigma_p = 7{,}500$

Your suite of prizes contains multiple prizes with values.

$$\text{Prize}_i \sim N(\mu_i, \sigma_i)$$

$$\text{Total price} = \text{Prize}_1 + \text{Prize}_2 + \underbrace{\epsilon}_{\text{error term}}$$

↳ Current observation

I dea: Update our belief in total price given that we observe $Prize_1$ and $Prize_2$ and have guesses about their values.

PRIZES:

1. A wonderful trip to Toronto, Canada!
2. A lovely new snowblower!

Step 1: Do run-off-the mill Bayesian analysis to estimate posterior suite price.

Step 2: Incorporate loss function to choose "best" guess of price.
↳ within $250 below the true price.

a) Define a loss function that analyzes overbidding greater than bidding under $250 greater than bidding under within $250.

b) Calculate expected loss for a grid of bids using Monte Carlo + posterior suite price.

c) Determine the risk you are willing to take (according to the risk parameter defined for this problem) and identify the lowest risk bid.

Side note:
You could come up w/ a better loss function if instead of an arbitrary risk factor, you target profit. (+ game theory)

Note that Bayesian modeling + fitting was first

↓

Define loss function

↓

Minimize expected loss using Monte Carlo

Example. Financial Prediction

Aim: Predict a stock's change

An interesting point: squared loss treats over + under predicting the same.

In the case of investing or shorting, we should take into account the sign difference of our prediction $\hat{y}$ and observed value y.

$$L(y,\hat{y}) = \begin{cases} \alpha \hat{y}^2 - \text{sign}(y)\hat{y} + y & \text{if } y \cdot \hat{y} < 0 \\ |y - \hat{y}| & \text{if } y \cdot \hat{y} \geq 0 \end{cases}$$

$y \cdot \hat{y} < 0$ if you predicted the wrong direction of the stock.

Model: we look at data of returns vs trading signal

$R = \alpha + \beta x + \varepsilon$

$\varepsilon \sim N(0, \sigma^2)$

$\beta \sim N(0, 100)$

$\alpha \sim N(0, 100)$

$\sigma^2 \sim U(0, 100)$

$\xrightarrow{\text{mcmc}}$ $R_i(x) = \alpha_i + \beta_i x + \varepsilon$

Posterior distribution on this regression model.

- For each trading signal x, we want the return (prediction) to minimize $\mathbb{E}[L(R(x), r)]$. Use Monte Carlo to identify the r across x.

- Our loss penalizes heavily for guessing the incorrect direction of return. So our Bayesian prediction is "pulled" to 0 when the trade signal is close to 0.