Network of Loyalty Programs: A Sequential Formation

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Abstract

By forming partnerships with hotel chains and airlines, major credit card issuers in the U.S. allow customers to transfer points from their loyalty programs to the partners'. Hotel chains form similar partnerships with airlines, and airlines with other airlines. Viewing their partnerships as a network and using a sequential network formation model, this article studies how credit card issuers and hotel chains strategically choose to add, delete, or maintain partnerships with airlines. Using a novel dataset that involves 3 credit card issuers, 7 hotel chains, and 43 airlines from 2014 to 2018, the estimation result suggests that a credit card issuer is more likely to form a partnership with an airline that (1) is a partner of another credit card issuer (2) is a partner of its hotel chain partner (3) better complements its existing portfolio of airline partners, after accounting for key characteristics of firms. The first two results are similar when hotel chains choose airline partners.

Keywords: Network formation, loyalty program, credit card

JEL codes: L14, L21, L80

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1 Introduction

A policy discussion article by the Federal Reserve Bank of Boston points out that credit card rewards transfer wealth from the poor to the rich, as credit card rewards received per dollar spent is positively associated with income (Schuh et al., 2010). All consumers share the cost of credit card rewards, as merchants typically attach the same price tag to credit card purchases and cash purchases. Yet, credit card issuers reward their customers in the form of cashback or points in their loyalty programs.

Major credit card issuers in the U.S. operate loyalty programs to reward customers. Customers may earn points in the loyalty programs by signing up for credit card products or making purchases using them. An example is the Membership Rewards program of the American Express Company (AMEX):

> "...through our Membership Rewards program we have partnered with businesses in many industries, including the airline industry, to offer benefits to Card Member participants."
>
> American Express Company (2014-2018).

Through partners, credit card issuers allow customers to redeem the points for a variety of rewards, such as cashback, gift card, and travel reward. The rewards, especially travel rewards, are a significant source of product differentiation. By forming partnerships with hotel chains and airlines, credit card issuers allow customers to transfer points from its loyalty program to the partners'. Hotel chains also form partnerships with airlines, and airlines form similar partnerships with other airlines.

The network of loyalty programs puts together partnerships across credit card issuers, hotel chains, and airlines. Using a sequential strategic network formation process, this article studies how credit card issuers and hotel chains choose their airline partners. This article studies how network-based relationships affect the choice of airline partners, above and beyond the characteristics of firms. The main focus is on credit card issuers.

Figure 1 illustrates two examples of network-based relationships. In sub-figure (i), C1 and C2 are credit card issuers, and A is an airline. C2 could be more or likely to choose A as a partner because A is a partner of its competitor C1. In sub-figure (ii), C is a credit card issuer, H is a hotel chain, and A is an airline. C could be more or less likely to choose A as a partner because A is a partner of it partner hotel chain H.

For a credit card issuer, the marginal benefit of adding, deleting, or maintaining partnership with an airline could depend not only on the characteristics of the airline but also on the combined characteristics of its existing portfolio of airline partners. For example, how adding an airline partner complements the existing portfolio of airline partners could explain the choice to add or not, above and beyond the standalone characteristics of the airline. By forming partnerships with airlines, a credit card issuer enables customers to transfer points from it's loyalty program to the partner airlines' and then redeem for reward flights. Different airlines operate over different flight routes and have cost advantages in different geographic regions. Consequently, their loyalty programs offer reward flights to different geographic regions via different flight routes at different costs, where the costs are in terms of loyalty points or "miles". However, airlines are not completely different from each other. For example, KE (Korean Air) and OZ (Asiana Airlines) both have the Incheon International Airport as their hub and offer reward flights between the U.S. and South Korea. The marginal benefit of adding OZ as partner could be small if a credit card issuer already has KE as a partner.

A sequential network formation process similar to Christakis et al. (2010) describes the partnership formations. The key feature is that there exists an unknown sequence of bilateral meetings via which pairs of agents may form partnerships. All eligible pairs of agents meet exactly once. The outcome of each meeting is public information and affects subsequent meetings. Agents are assumed to be myopic, meaning that meeting outcome depends on the current state of the network at the time of the meeting.

Unlike a social network, the network of loyalty programs involves three classes of agents: credit card issuers, hotel chains, and airlines. Partnerships between certain classes are structurally impossible. For example, a credit card issuer never has a partnership with another credit card issuer. Moreover, a partnership is not necessarily mutual. For example, DL (Delta Air Lines) is a partner of AMEX because points can be transferred from AMEX's loyalty program to DL's. However, points cannot be transferred in the reverse direction. To accommodate such structure, the network splitted into four subnetworks: $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$. $\mathbf{D}_1$ describes partnerships between credit card issuers and airlines, $\mathbf{D}_2$ describes partnerships between hotel chains and airlines, $\mathbf{D}_3$ describes partnerships between credit card issuers and hotel chains, and $\mathbf{D}_4$ describes partnerships between airlines. The formation of $\mathbf{D}_1$ and $\mathbf{D}_2$ are modeled while taking $\mathbf{D}_3$ and $\mathbf{D}_4$ as exogenous.

Each of subnetworks $\mathbf{D}_1$ and $\mathbf{D}_2$ involves two classes of agents, and they play asymmetric roles as *choosers* and *bidders*. For $\mathbf{D}_1$, credit card issuers are choosers and airlines are bidders. For $\mathbf{D}_2$, hotel chains are choosers and airlines are bidders. Bidders initially send take-it-or-leave-it bids to choosers. Afterwards, bilateral meetings between choosers and bidders occur sequentially according to an unknown sequence of meetings. The sequence of meetings is a chronological order of meetings over all eligible pairs of agents in

$\mathbf{D}_1$ and $\mathbf{D}_2$ combined. In each meeting, the chooser either accepts or rejects the bidder's bid by optimizing over the current state of $\mathbf{D}_1, \mathbf{D}_2$, and variables that are not modeled, including $\mathbf{D}_3, \mathbf{D}_4$, and the characteristics of firms. As an outcome, the chooser either adds, removes, or maintains the bidder as a partner. The outcome updates the current state of $\mathbf{D}_1$ or $\mathbf{D}_2$ for the next meeting. All eligible (chooser, bidder) pairs meet exactly once.

Estimation involves two steps. The first step constructs bids for (credit card issuer, airline) and (hotel chain, airline) pairs without partnership, as bid are observed only for pairs with partnership. A linear regression model is fitted using observed bids and cubic polynomials of the characteristics of firms, and then the unobserved bids are predicted. Treating constructed bids as data, the second step utilizes the method of Christakis et al. (2010) to estimate model parameters.

Using a novel dataset that involves 3 credit card issuers, 7 hotel chains, and 43 airlines from 2014 to 2018, the estimation result suggests

1. A credit card issuer is more likely to add an airline partner that is a partner of another credit card issuer than otherwise,
2. A credit card issuer is more likely to add an airline partner that is a partner of its hotel chain partner(s) than otherwise,
3. A credit card issuer is more likely to add an airline partner that better complements its existing portfolio of airline partners than otherwise, so that reward flights offered by its loyalty program are more cost-effective and cover more diverse flight routes,

after accounting for key characteristics of firms. The first two results are similar when hotel chains choose airline partners.

The rest of this article is organized as the following. Section 2 discusses related research. Section 3 describes the industry and incentives in forming partnerships. Section 4 describes the network of loyalty programs and splitting into subnetworks. Section 5 describes the model, and section 6 explains the estimation method. Section 7 discusses the data and presents the estimation result. Section 8 concludes.

2 Related Research

Researchers often model strategic network formation as a collection of pairwise linkages, where a link between a pair of agents forms depending on their characteristics and possibly the characteristics of other agents. Researchers often regard the observed state of the network as an equilibrium outcome, using the pairwise stability of Jackson and Wolinsky (1996) as the equilibrium condition. The pairwise stability extends Nash equilibrium to accommodate pairwise interactions, such that for each pair, neither one of the agents can be made better off by modifying the linkage between the pair.

Research that employs the pairwise stability views strategic network formation as a simultaneous-move game. Simultaneity leads to multiple equilibria, which complicates estimation. Tamer (2003) explains that naively using a regression model may result in biased estimation in the presence of multiple equilibria. de Paula et al. (2015) and Sheng (2020) separately presents a partial identification method for network formation models based on subnetworks. Jia (2008) utilizes Tarski (1955)'s lattice theory to restrict the set of Nash equilibria in a simultaneous-move game, where Walmart and K-Mart strategically choose their store locations. Nishida (2015) extends the method of Jia (2008) to study strategic location choices by the convenience-store industry in Japan. Miyauchi (2016) extends the method of Jia (2008) and presents an identification method for analyzing the formation of pairwise stable networks. Lee and Fong (2013) develops a two-stage model of strategic network formation, where the first stage determines potential partners and the second stage models link formations as bilateral Nash bargaining games.

Christakis et al. (2010) and Snijders et al. (2010) separately presents a model of sequential network formation, in which the network is formed via some unknown sequence of meetings between myopic agents. By construction, the equilibrium is uniquely pinned down, conditional on a sequence of meetings. However, the model parameters cannot be directly estimated because the sequence of meetings is unknown. They employ Bayesian methods to sample the sequence of meetings and estimate the model parameters using Markov Chain Monte Carlo. This article is built upon their framework but with focus on the strategic formation of the portfolio of partners, with agents playing asymmetric roles in the network.

Researchers of strategic business management have studied incentives in partnership formation between firms. Several notable developments include the following. Gulati (1995) analyzes how the network of firms affects the formation of alliances using a panel data of partnerships between firms . Chung et al. (1999) studies how complementarity, status similarity, and the network of firms affect alliance formation across investment banks in the U.S. Rothaermel and Boeker (2008) studies the role of complementarity and status similarity in the formation of alliances using a network of pharmaceutical firms. Lin et al. (2009) studies the motives in alliance formation based on how complementarity, status, and the network of firms affect the performance of firms.

3 Industry Background

Major credit card issuers in the U.S. operate loyalty programs to reward customers. For example, AMEX (American Express Company), CITI (Citibank), and JPMC (J.P. Morgan Chase) operate Membership Rewards, ThankYou Rewards, and Ultimate Rewards, respectively. These loyalty programs are associated with their flagship credit card products, and customers may earn points in the loyalty program (loyalty points) by signing up for or making purchases using the products. The loyalty program and the associated point are different for each credit card issuer.

The loyalty program of a credit card issuer is an important marketing tool. Wirtz et al. (2007) finds that the attractiveness of a credit card issuer's loyalty program has a positive effect on the share of wallet, which is the share of credit card purchases made using the credit card issuer's products. In 2015, 55 percent of U.S. consumers chose rewards as the most attractive credit card feature, and the percentage rose to 79 percent in 2018[1] (Total Systems Services, 2016-2018). The fact that credit card issuers sharply increased expenditures in their loyalty programs further support the importance. In the fourth quarter of 2016, JPMC spent up to $300 mil. on Ultimate Rewards[2]. The expenditures of AMEX on Membership Rewards increased from $6.8 bil. in 2016 to $7.6 bil in 2017, and then to $9.7 bil. in 2018. For 2017 and 2018, these expenditures accounted for a third of AMEX's total expenses. At the end of 2018, AMEX's liability for unclaimed points in Membership Rewards was $8.4 bil. (American Express Company, 2014-2018).

Via partners, credit card issuers offer customers a variety of redemption options for loyalty points, such as travel reward and gift card, in addition to cashback. The portfolio of partners, especially hotel chains and airlines, is an important source of product differentiation for credit card issuers. Hotel chains and airlines also operate their own loyalty programs. Redeeming a credit card issuer's loyalty points for travel reward is typically done by transferring the points to partners' loyalty programs, according to pre-specified conversion

[1] In 2018, interest rate was ranked second at 67 percent, and card brand was ranked third at 55 percent.

[2] Jennifer Surane and Hugh Son. (2016). "Dimon Says New Sapphire Card Cuts Profit by Up to $300 Million in Quarter." *Bloomberg.* December 6. https://www.bloomberg.com/news/articles/2016-12-06/dimon-says-new-card-cuts-profit-by-up-to-300-million-in-quarter.

ratio. For example, customers of AMEX can transfer loyalty points from AMEX to DL (Delta Air Lines) with 1:1 conversion ratio and then redeem for reward flights offered by DL. Different credit card issuers are partnered with different sets partner firms. In addition to conversion ratios, which firms it is partnered with affects the attractiveness of the travel rewards offered by a credit card issuer and thus its loyalty program.

On the other hand, redeeming points for non-travel reward is hardly different from cashback. For example, customers of AMEX, CITI, and JPMC may use loyalty points to purchase goods and services sold at Amazon.com[3], with cents-per-point conversion rates of 0.7, 0.8, and 0.8, respectively. The corresponding the cents-per-point conversion rates for cashback are 0.6, 0.5, and 1, respectively. These rates are prespecified, and redeeming for non-travel rewards is essentially the same as redeeming for cash, possibly with a small discount. Unlike travel rewards, there is not much room for product differentiation for these types of rewards.

In this article, a *partnership* between two firms means points can be transferred between their loyalty programs. A partnership is not necessarily exclusive, as credit card issuers and hotel chains share common airline partners. Table 1 reports the chronological changes of credit card issuers' partners from Q2 2013 to Q3 2019. To facilitate the transfer of points, firms purchase points from their partners. Credit card issuers purchase points from partner hotel chains and airlines. Hotel chains also purchase points from airlines to facilitate the transfer of points to their partner airlines. From 2014 to 2015, AA (American Airlines) earned 1.3-1.6 cents per point sold to non-airline partners (American Airlines, 2014-2018). The sales of points is a significant source of revenue for airlines. In 2017, AA earned \$2.2 bil. revenue from selling points, which accounted for about 5.2 percent of the airline's total operating revenue (American Airlines, 2014-2018) for that year. In the same year, UA (United Airlines) earned approximately \$1.2 bil. revenue from selling points, and it accounted for about 3.2 percent of the airline's total operating revenue for that year (United Airlines, 2014-2018).

Partnership as defined in this article is a weaker relationship than *co-brand partnership*. An example of co-brand partnership is that AMEX issues credit card products under the brand name of DL. Joint with DL, AMEX offers its customers complementary services, including free checked baggage and airport lounge access while using DL's flight services. Moreover, customers can earn points in DL's loyalty program by making purchases using the co-brand credit card products. Such co-brand partnership is mutually beneficial for both firms. DL gains from discounted credit card processing fees and by selling its loyalty points to

[3]One may also view this as redeeming for gift cards at Amazon.com.

AMEX[4]. From the co-brand partnership, DL gained \$3.4 bil. in 2018, and it expects the annual benefit to grow to \$7 bil. by 2023[5]. AMEX also gains by appealing to customers of DL, expanding its customer base. In fact, AMEX reported that its co-brand credit card products with DL accounted for 8 percent of its credit card purchases and 21 percent of its total outstanding credit card loans in 2018 (American Express Company, 2014-2018). Co-brand partnership is almost always exclusive[6] and typically lasts for more than ten years. Co-brand partnership implies partnership. In the empirical analysis, partnerships that are implied by co-brand partnerships are taken as exogenous.

4 Network of Loyalty Programs

The network of loyalty programs presented in this article describes partnerships among 3 credit card issuers, 7 hotel chains, and 43 airlines. Their names are abbreviations are listed in 2. Figure 2 is a snapshot of the network observed in November 2018[7]. Gold nodes are credit card issuers, teal nodes are hotel chains, and white nodes are airlines. An arrow from node i to node j indicates partnership. It is synonymous to a *link* from i to j. There is an *indirect link* from i to j if there is a path from i to j via 2 or more links. $\mathbf{D}$ denotes the matrix that contains the linkage information across all those firms. $\mathbf{D}_{ij} = [\mathbf{D}_{ij}] = 1$ if there is a link from i to j and 0 if not. $\mathbf{D}_{ij}$ and $\mathbf{D}_{ji}$ are not necessarily equal because directions matter. In this article, *network* or *network of loyalty program* and $\mathbf{D}$ are interchangeable.

If at least one of i or j is not an airline, a link from i to j means points can be transferred from i's loyalty program to j's. If i and j are both airlines, a link from i to j means i's loyalty program can used to redeem for a flight offered by j. There is a link from i to j and from j to i if they are members of the same airline alliance. There are also bilateral partnerships that gives a link between airlines. In figure 2, nodes are positioned closer to each other if there is a higher frequency of direct and indirect links between them. The cluster of airlines at the top is Oneworld, lower-left is Star Alliance, and lower-right is SkyTeam. It indicates AMEX is positioned close to SkyTeam, while JPMC and CITI are positioned close to Star

[4]Whenever customers of the co-branded credit card products receive points in DL's loyalty program, AMEX pays DL for the points.

[5]Delta Air Lines. (2019). "American Express and Delta Renew Industry-Leading Partnership, Lay Foundation to Continue Innovating Customer Benefits." April 2. https://news.delta.com/american-express-and-delta-renew-industry-leading-partnership-lay-foundation-continue-innovating.

[6]An exception is that American Airlines has co-brand partnerships with both Citibank and Barclays US.

[7]There are 3 credit card issuers, 6 hotel chains, and 41 airlines in this figure because MAR completed its acquisition of SPG, and their loyalty programs were integrated in 2017. AB filed bankruptcy in 2017. SA was omitted because it significantly shrank in size in 2017 due to financial hardships.

Alliance and Oneworld.

Figure 3 is a close-up view with only a subset of the firms. The number attached to a link is the conversion ratio of loyalty points. For example, the conversion ratio from HLT to AC is 0.1, indicating customers of HLT can transfer 1 point in HLT's loyalty program to obtain 0.1 points in AC's loyalty program. For links between airlines, the number is always set to 1. Conversion ratios obviously do not exist for firms with a link.

Figure 3 illustrates three critical features of the network of loyalty programs. First, except for airlines, there are no links between firms within the same industry sector, indicating those firms within the same industry are competitors with no partnerships or alliances. Second, there are no links towards credit card issuers, meaning points can never be transferred into the loyalty programs of credit card issuers. Third, there are no links from airlines to hotel chains[8]. To model the formation of the network of loyalty programs, accounting for such structurally impossible linkages is imperative.

Subnetworks

In figure 4, the network in figure 3 is split into four subnetworks. The subnetwork in sub-figure (i) involves credit card issuers and airlines, (ii) involves hotel chains and airlines, (iii) involves between credit card issuers and hotel chains, and (iv) involves only airlines. Subnetworks in (i), (ii), (iii) exhibit bipartite structure, as there are two groups of nodes and links are directed only from one group to the other. To be precise, (i) only describes linkages from credit card issuers to airlines are possible, (ii) only describes linkages from hotel chains to airlines are possible, and (iii) only describes linkages from credit card issuers to hotel chains. Let $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ denote the subnetworks of $\mathbf{D}$ that correspond to types (i), (ii), (iii), (iv), respectively. By modeling subnetworks $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ and putting them together, one can model the whole network $\mathbf{D}$ while accounting for structurally impossible links. As a reminder, all elements of matrices $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ are binary, as matrix $\mathbf{D}$.

This article models the formation of only $\mathbf{D}_1$ and $\mathbf{D}_2$, while taking $\mathbf{D}_3$ and $\mathbf{D}_4$ as exogenous. The main focus is on $\mathbf{D}_1$, which describes partnerships between credit card issuers and airlines. Modeling $\mathbf{D}_2$ is still necessary to allow the formation of $\mathbf{D}_1$ and $\mathbf{D}_2$ to depend on each other. $\mathbf{D}_3$ is taken as exogenous because partnerships between credit card issuers and hotel chains are determined by co-brand partnerships, which

[8]There are a few exceptions. Points can be transferred from the loyalty programs of a couple of airlines to HLT. These are ignored.

are almost always exclusive partnerships that last over a decade. The last paragraph of section 3 describes co-brand partnership in more detail. $\mathbf{D}_4$ is taken as exogenous because the vast majority of partnerships between airlines are tied to airline alliances, which stayed unchanged between 2014 and 2018. Moreover, the formation of partnerships between airlines is complex as it involves strategic motives over flight routes and regulations by domestic and foreign agencies.

5 Model

Let $\mathbf{D}^0$ denote the initial state of the network, and $\mathbf{D}^1$ the terminal state. The model describes the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$ as outcomes of bilateral meetings of agents. Agents play asymmetric roles as *bidders* and *choosers*. Initially, all bidders submit take-it-or-leave-it bids to all choosers, where bids are private information. Afterwards, bilateral meetings between bidders and choosers occur sequentially according to a *sequence of meetings*. In each meeting, the chooser either accepts or rejects the bidder's bid, and it is the outcome of the meeting. Accepting of rejecting the bid has different implications on $\mathbf{D}^1$ depending on whether the (bidder, chooser) pair had a link in $\mathbf{D}^0$ or not. If they had a link in $\mathbf{D}^0$, then accepting the bid implies the link is maintained in $\mathbf{D}^1$, and rejecting implies the link is severed in $\mathbf{D}^1$. If they did not have a link in $\mathbf{D}^0$, then accepting the bid implies a new link is created in $\mathbf{D}^1$, and rejecting implies they still don't have a link in $\mathbf{D}^1$. Therefore, there are four possible outcomes for each meeting.

Key model assumptions are (1) all eligible (bidder, chooser) pairs meet exactly once, where the sequence of meetings is unknown to the econometrician and the agents, (2) agents are myopic, in the sense that the outcome of a meeting does not depend on expectations on the outcomes of future meetings, (3) the outcome of a meeting immediately becomes public information, which may affect the outcomes of subsequent meetings, and (4) bids are determined non-strategically, meaning that bids are independent conditional on key performance indicators (KPIs) of the bidder and the chooser. The model is built upon the sequential social network formation of Christakis et al. (2010). Key distinction is that the network is partitioned into subnetworks to accommodate distinct classes of agents: credit card issuers, hotel chains, and airlines. The use of subnetworks accounts for structurally impossible linkages and also assigns asymmetric roles to agents in generating the outcomes of meetings. Conforming to institutional facts, the formation of subnetworks $\mathbf{D}_1$ and $\mathbf{D}_2$ are modeled while taking $\mathbf{D}_3$ and $\mathbf{D}_4$ are exogenous. The remainder of this section describes the objective function of the choosers, bids, the sequence of meetings, and then puts them together to describe the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$.

Chooser

Let $\mathbb{I}$ denote the set of credit card issuers and $\mathbb{K}$ the set of airlines. After receiving bids from all airlines, $i \in \mathbb{I}$ strategically *chooses* a portfolio of airline partners, $P_i \subseteq \mathbb{K}$, to maximize its objective function. The objective function of i has the following form:

$$U\left(P_i; \alpha_i, b_i, w_i, \mathbf{A}, \mathbf{D}_{-i}, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_i\right) = \alpha_i + g\left(P_i, b_i, w_i, \mathbf{A}, \mathbf{D}_{1,-i}, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_i\right) + \sum_{k \in P_i} \epsilon_{ik}. \tag{1}$$

$\alpha_i \in \mathbb{R}$ denotes fixed effect specific to chooser i. $b_i := \left(b_{ik}\right)_{k \in \mathbb{K}}$, where $b_{ik} \in \mathbb{R}_{++}$ denotes the *bid* that i receives from airline k. $w_i := \left(w_{ik}\right)_{k \in \mathbb{K}}$, where $w_{ik} \in \mathbb{R}_{++}$ denotes the *potential conversion ratio* from i to k. The next subsection discusses bid and potential conversion ratio in greater detail. Matrix $\mathbf{A}$ contains key performance indicators (KPIs) of all airlines. A_k', the k^{th} row of $\mathbf{A}$, contains key performance indicators of airline k. $\mathbf{D}_{1,-i}$ denotes subnetwork $\mathbf{D}_1$ after removing the elements that involve i. Combining the information in P_i and $\mathbf{D}_{1,-i}$ yields $\mathbf{D}_1$. Thus, combining the information in $P_i, \mathbf{D}_{1,-i}, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ yields the full network $\mathbf{D}$. β denotes the vector of parameters for the credit card issuer's objective function. Finally, $\epsilon_i := \left(\epsilon_{ik}\right)_{k \in \mathbb{K}}$, where $\epsilon_{ik} \in \mathbb{R}$ denotes unobserved heterogeneity of credit card issuer i towards airline k.

Assumption 1. *For all $i \in \mathbb{I}$ and $k \in \mathbb{K}$,*

$$\epsilon_{ik} \big| \alpha_i, b_i, w_i, \mathbf{A}, \mathbf{D} \sim i.i.d. \tag{2}$$

In words, equation (2) states that unobserved heterogeneity across (credit card issuer, airline) pairs are independent and identically distributed, conditional on the fixed effect specific to the credit card issuer, the bids it receives, the potential conversion ratios of points from its loyalty program to airlines, the KPIs of all airlines, and the state of the whole network. I use the standard logistic distribution for the empirical analysis in section 7.

Function $g(\cdot)$ takes the following form:

$$\begin{aligned} g\left(P_i, b_i, w_i, \mathbf{A}, \mathbf{D}_{1,-i}, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4, \beta\right) = {} & \beta_1 Competitor\left(P_i, \mathbf{D}_{1,-i}\right) + \beta_2 Transitivity\left(P_i, \mathbf{D}_2, \mathbf{D}_3\right) \\ & + \beta_3 Mileage\left(P_i, w_i, \mathbf{D}_4\right) + \beta_4 Route\left(P_i, \mathbf{D}_4\right) + \beta_5 GeoHub\left(P_i\right) \\ & + \beta_6 \left|P_i\right| + \beta_7' Performance\left(P_i, b_i, \mathbf{A}\right), \end{aligned} \tag{3}$$

where $\beta = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7')'$.

The first two components capture how i's choice of airline partners is influenced by their partnerships with other credit card issuers and hotel chains. *Competitor* gives the number of i's airline partners that are also partners of at least one credit card issuer other than i. That is,

$$Competitor\,(P_i, \mathbf{D}_{1,-i}) = \left|\left\{k \in P_i : [\mathbf{D}_{1,-i}]_{\tilde{i}k} = 1, \text{ for some } \tilde{i} \neq i\right\}\right|. \tag{4}$$

Because subnetwork $\mathbf{D}_1$ describes linkages from credit card issuers to airlines, $[\mathbf{D}_{1,-i}]_{\tilde{i}k} = 1$ indicates airline k is a partner of credit card issuer $\tilde{i}$. Positive (negative) β_1 would suggest that i tend to value (devalue) airline partners that are also partners of other credit card issuers, which are competitors of i. *Transitivity* gives the number of i's airline partners that are also partners of at least one of i's hotel chain partners. That is,

$$Transitivity\,(P_i, \mathbf{D}_2, \mathbf{D}_3) = \left|\left\{k \in P_i : [\mathbf{D}_2]_{jk} = 1, \text{ for some } j \text{ such that } [\mathbf{D}_3]_{ij} = 1\right\}\right|. \tag{5}$$

Subnetwork $\mathbf{D}_2$ describes linkages from hotel chains to airlines, and $\mathbf{D}_3$ describes linkages from credit card issuers to hotel chains. Thus $[\mathbf{D}_3]_{ij} = 1$ and $[\mathbf{D}_2]_{jk} = 1$ indicates there is an indirect path from i to k via j[9]. Positive (negative) β_2 would suggest that i tend to value (devalue) airline partners that are also partners of i's hotel chain partners. This component captures a measure of link transitivity across credit card issuers, hotel chains, and airlines. β_1 and β_2 capture how network linkages affect partnership formation, and they are parameters of primary interest.

The next three components capture how the choice of i's airline partners is influenced by their direct benefits to i's loyalty program. Having airline partners enables customers of i to transfer points in i's loyalty program to those airlines and then redeem for reward flights. Portfolio of airline partners P_i yields smaller (larger) value of $Mileage$ if the available reward flights are overall more (less) cost-efficient, in terms of the number of points necessary for redemption. In other words, a smaller value of $Mileage$ implies customers of i can use a smaller average amount of loyalty points, in real values, to redeem for a reward flight, where the amount of loyalty points is averaged over the flight routes covered by i's loyalty program. P_i yields a larger (smaller) value of *Route,* if the available reward flights cover more (less) diverse flight routes. For example, suppose P_i covers only one flight route, between JFK (John. F. Kennedy Airport) and LHR (Heathrow Airport). If a different portfolio P_i' covers that route and another route between LAX (Los

[9] Points in i's loyalty program can be transferred to j's and then from j's to k's loyalty program.

Angeles International Airport) and LHR, then P_i' yields a larger value of $Route$ than P_i. $GeoHub$ counts the number of distinct airline hub locations. A larger (smaller) value of $GeoHub$ indicates P_i is associated with larger (smaller) geographic diverseness, in terms of hub locations of partner airlines. For example, suppose $P_i = \{\text{DL, KE}\}$. In terms of geographic diverseness, P_i is associated with North America (DL; Delta Air Lines) and Far East Asia (KE; Korean Air). Adding NH (All Nippon Airways) to P_i does not change the value of $GeoHub$ because NH's hub is in Far East Asia (Tokyo). More details on $Mileage$, $Route$, and $Geohub$ are documented in the online appendix.

$|P_i|$ counts the size of the portfolio of airline partners P_i. Although the objective function does not explicitly specify the cost and benefit associated with the number of airline partners, including $|P_i|$ parsimoniously accounts for it. $Performance\,(P_i, b_i, \mathbf{A})$ is equal to

$$\sum_{k \in P_i} \frac{b_{ik}}{\sum_{k \in P_i} b_{ik}} A_k, \tag{6}$$

which is bid-weighted average of partner airlines' KPIs, with larger weights assigned to airlines that submitted larger bids to i. Large b_{ik} means k's bid is a "good deal" for i. Due to variable definitions, the associated coefficients β_6 and β_7 do not have straightforward interpretations. The last two components serve as controls.

The objective function is similar for subnetwork $\mathbf{D}_2$, which describes linkages from hotel chains to airlines. Hotel chain j's objective function is

$$V\left(P_j; \gamma_j, b_j, w_j, \mathbf{A}, \mathbf{D}_1, \mathbf{D}_{2,-j}, \mathbf{D}_3, \mathbf{D}_4, \delta, \epsilon_j\right) = \gamma_j + h\left(P_j, b_j, w_j, \mathbf{A}, \mathbf{D}_1, \mathbf{D}_{2,-j}, \mathbf{D}_3, \mathbf{D}_4, \delta\right) + \sum_{k \in P_j} \epsilon_{jk}, \tag{7}$$

where $P_j \subseteq \mathbb{K}$ denotes j's portfolio of airline partners. δ denotes the vector of parameters for subnetwork $\mathbf{D}_2$. $\theta := (\beta', \delta')'$ puts together the parameters for subnetworks $\mathbf{D}_1$ and $\mathbf{D}_2$.

Bid

Customers of credit card issuer i can transfer points from i's loyalty program to airline k's loyalty program if k is a partner of i. Points can be transferred because i purchases k's points, based on pre-determined price per point. The bid that i receives from k (or k sends to i) characterizes the price per point. $b_{ik} \in \mathbb{R}_{++}$ denotes the bid, and it is given by

$$b_{ik} = \frac{w_{ik}}{v_i/v_k}. \tag{8}$$

w_{ik} is potential conversion ratio from i to k, which is the amount of points in k's loyalty program that can be obtained per 1 point in i's loyalty program. It is *potential* because points can be transferred only if i accepts k's bid (k is a partner of i). Thus, a larger w_{ik} implies one can obtain more points in k's loyalty program per 1 point in i's loyalty program. v_i and v_k denote the real value[10] of i's and k's points, respectively. Thus, a larger v_i/v_k implies points in i's loyalty program is more valuable than k's. Equation (8) describes bid as potential conversion ratio, after accounting for differences in relative values of points. For i, a larger b_{ik} is more favorable because 1 point in i's loyalty program can obtain points in k's loyalty that are more than the relative values of points.

w_{ik} is observed by the econometrician only when k is a partner of i. Thus b_{ik} is observed (can be constructed from observed variables) only if i accepts k's bid, and all the rejected bids are unobserved. The unobserved bids are constructed using a linear regression model

$$b_{ik} = \eta_0 + \eta_1' C_i + \eta_2' A_k + \epsilon_{ki}. \tag{9}$$

C_i and A_k denotes KPIs of credit card issuer i and airline k, respectively. ϵ_{ki} denotes unobserved heterogeneity of airline k specific to credit card issuer i. Note that ϵ_{ki} is different from ϵ_{ik}, which appeared in the previous subsection.

Assumption 2. *For all $i \in \mathbb{I}$ and $k \in \mathbb{K}$,*

$$\epsilon_{ki} \Big| C_i, A_k \sim \textit{i.i.d.} \tag{10}$$

This assumption states bidders are non-strategic agents, as bids are determined only at pairwise level. An implication is that bids do not depend on the network linkages $\mathbf{D}$, conditional on their KPIs.

After constructing the unobserved bids, corresponding potential conversion ratios are constructed using equation 9. The bid and potential conversion ratio associated with subnetwork $\mathbf{D}_2$ (airlines send bids to hotel chains) are similar with credit card issuer i replaced by hotel chain j.

[10] Real values are calculated as an average over the goods and services that can be acquired by spending loyalty points. A larger value means it requires less amount of points to obtain comparable goods and services. The online appendix documents how the real values were computed.

Sequence of Meetings

After bidders submits bids to choosers, bilateral meetings between bidders and choosers occur according to a sequence of meetings. In each meeting, the chooser either accepts or rejects the bidder's bid to maximize its objective function. Similar to Christakis et al. (2010), all eligible (chooser, bidder) pairs meet exactly once.

Let

$$S_1 := \left[\left(i^1, k_1^1\right), \left(i^2, k_1^2\right), \ldots, \left(i^{M_1}, k_1^{M_1}\right)\right] \tag{11}$$

$$S_2 := \left[\left(j^1, k_2^1\right), \left(j^2, k_2^2\right), \ldots, \left(j^{M_2}, k_2^{M_2}\right)\right]. \tag{12}$$

S_1 denotes the sequence of meetings for subnetwork $\mathbf{D}_1$. For $(i^{m_1}, k_1^{m_1})$, the superscript m indicates it is the m_1^{th} meeting for $\mathbf{D}_1$, where credit card issuer i^{m_1} is the chooser and airline $k_1^{m_1}$ is the bidder. $M_1 := |\mathbb{I}| \times |\mathbb{K}|$ is the number of meeting for $\mathbf{D}_1$. S_2 denotes similar sequence of meeting for subnetwork $\mathbf{D}_2$.

By combining S_1 and S_2, we can define the total sequence of meetings S over $M := M_1 + M_2$ meetings. Note that the meeting $(i^{m_1}, k_1^{m_1})$ can happen after the meeting $(j^{m_2}, k_2^{m_2})$, and vice versa. Because subnetworks $\mathbf{D}_3$ and $\mathbf{D}_4$ are taken as exogenous, S completely specifies the chronological order of meetings that yields the transition from the initial state of the network $\mathbf{D}^0$ to the terminal state $\mathbf{D}^1$. For $m = 1, 2, \ldots, M$, $\mathbf{D}^{0,m}$ denotes the state of the network at the beginning of the m^{th} meeting, and $\mathbf{D}^{0,m+1}$ the state at the end of the m^{th} meeting. Thus, the initial state is $\mathbf{D}^0 = \mathbf{D}^{0,1}$, and the terminal state is $\mathbf{D}^1 = \mathbf{D}^{0,M+1}$. For subnetworks $\mathbf{D}_1$ and $\mathbf{D}_2$, $\mathbf{D}_1^{0,m}$ and $\mathbf{D}_2^{0,m}$ are defined similarly.

Without loss of generality, let (i^m, k^m) be the m^{th} meeting in S. Let $P_{i^m}^m$ and $P_{i^m}^{m+1}$ denote credit card issuer i^m's portfolio of ariline partners at the beginning and at the end of the m^{th} meeting, respectively. In other words, $P_{i^m}^{m+1}$ is $P_{i^m}^m$ after updating the outcome of the m^{th} meeting, in which i^m can add or remove k^m to its portfolio of airlines, or maintain status quo. $\boldsymbol{D}_{1,-i^m}^{0,m}$ denotes the state of subnetwork $\mathbf{D}_1$ at the beginning of the m^{th} meeting, after removing the elements that involve i^m. Thus, combining $\mathbf{D}_{1,-i^m}^{0,m}$ with

$P_{i^m}^m$ yields $\mathbf{D}_1^{0,m}$. Combining $\mathbf{D}_{1,-i^m}^{0,m}$ with $P_{i^m}^{m+1}$ yields $\mathbf{D}_1^{0,m+1}$. The outcome of the m^{th} meeting satisfies

$$U\left(P_{i^m}^{m+1}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \geq$$
$$U\left(P_{i^m}^m \cup \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \tag{13}$$
$$U\left(P_{i^m}^{m+1}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \geq$$
$$U\left(P_{i^m}^m \backslash \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right). \tag{14}$$

Equation (14) states that i^m cannot be made better off by adding k^m to its portfolio of airline partners (if k^m is not already a partner), conditional on the state of the network at the beginning of the m^{th} meeting and other things. Equation (14) states that i^m cannot be made better off by removing k^m from its portfolio (if k^m is already a partner) conditional on the state of the network at the beginning of the m^{th} meeting and other things. i^m is myopic because it only take into account the current state of the network at the m^{th} meeting, not expectations of future states. The conditions are similar for a meeting between a hotel chain and an airline, except that the outcome of their meeting updates subnetwork $\mathbf{D}_2$. To summarize, in each meeting, myopic chooser updates its portfolio of airline partners to maximize its objective function conditional the current state of the network. The outcome updates the state of the network, which affects the outcomes of subsequent meetings. The process ends with the outcome of the last meeting, which gives the terminal state of the network.

Dependence on Sequence of Meetings

A sequence of meetings lays out a chronological ordering of meetings that yield the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$. For a given (chooser, bidder) pair, the current state of the network may be different under a different sequence of meetings. As as consequence, the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$, and thus the model parameter θ, can uniquely pinned down only if we conditional on a sequence of meetings. Figure 5 in the appendix uses a small portion of the network of loyalty programs to illustrate how different values of θ could justify the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$.

In 2018, JPMC announced its partnership partnership with KE would end. It removes the link from JPMC to KE, altering the state of the network. A meeting between any chooser and bidder is subject to different states of the network depending on whether it occurs before or after the announcement. Sub-figure (i) illustrates the meeting between JPMC and NH (dashed arrow indicates NH is not a partner of JPMC at the time) if it happens before the announcement (red arrow emphasizes KE is a partner of JPMC at the time). Sub-figure (ii) illustrates the same meeting if it happens after the announcement (red dashed arrow emphasizes KE is not a partner of JPMC at the time). The meeting outcomes in (i) and (ii) could be different because adding NH as a partner could have different marginal contribution to JPMC's objective function depending on JPMC has KE as a partner or not[11]. Sub-figures (iii) and (iv) illustrates the meeting between AMEX and KE if it happens before or after the announcement, respectively. Because JPMC is a competitor of AMEX, adding KE as a partner could have different marginal contribution to AMEX's objective function depending on KE is a partner of JPMC or not.

IHG, which is a hotel chain partner of JPMC, added AC as a partner in 2016. Sub-figure (v) and (vi) illustrate the meeting between JPMC and AC if it happens before or after AC becomes a partner of IHG, respectively. Adding AC as a partner could have different marginal contribution to JPMC's objective function depending on whether AC is a partner of IHG (a hotel chain partner of JPMC) or not.

These examples demonstrate that different values of model parameters can justify the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$ depending on the sequence of meetings. Similar to Christakis et al. (2010), the true sequence of meetings is assumed to be unknown to the econometrician.

6 Estimation

Estimation is done in two stages. The first stage constructs bids for non-linked (chooser, bidder) pairs using a linear regression model, as given by equation (9), and then constructs potential conversion ratios for those pairs using equation (8). The constructed bids and potential conversion ratios are treated as data. The second stage estimates the model parameter θ using the method suggested by Christakis et al. (2010).

The model described in the previous section induces a likelihood function conditional on a sequence of meetings. Then two MCMC (Markov Chain Monte Carlo) iterations sample values of θ and sequences of meetings using the Metropolis-Hastings algorithm (Metropolis et al., 1953). There is feedback between the two iterations, such that a value of θ is sampled given a sequence of meetings, and then a sequence of meetings is sampled given that value of θ. Via the MCMC iterations, draws are made from the posterior distribution of θ unconditional on the sequence of meetings. The iterations end when the convergence criterion of Gelman and Rubin (1992) is satisfied.

[11] It is especially true because NH and KE operate similar flight routes. Both NH and KE have hubs in East Asia. NH's hub is at Tokyo, Japan. KE's hub is at Incheon, Korea.

This section describes the conditional likelihood function. The online appendix documents details of the MCMC iterations.

Marginal Contribution to Objective Function

As before, let (i^m, k^m) denote the m^{th} meeting in the sequence of meetings S. Define

$$\begin{aligned} \Delta U_+^m &:= U\left(P_{i^m}^m \cup \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \\ &\quad - U\left(P_{i^m}^m; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} \Delta U_-^m &:= U\left(P_{i^m}^m \setminus \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right) \\ &\quad - U\left(P_{i^m}^m; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta, \epsilon_{i^m}\right). \end{aligned} \tag{16}$$

In words, ΔU_+^m is the change in credit card issuer i^m's (chooser for the m^{th} meeting) objective function induced by adding k^m (bidder for the m^{th} meeting) to its portfolio of airline partners. If $\Delta U_+^m > 0$, then i^m is better off adding k^m to its portfolio of airline partners. Otherwise, i^m is better off maintaining the status quo, where k^m is not a partner. ΔU_-^m is the change in i^m's objective function induced by removing k^m from its portfolio of airline partners. If $\Delta U_-^m > 0$, then i^m is better off removing k^m from its portfolio of airline partners. Otherwise, i^m is better off maintaining the status quo, where k^m is not a partner.

Exploiting the additively separable specification of $U(\cdot)$, as given by equation (1), above expressions are equal to

$$\Delta U_+^m = \Delta g_+^m + \epsilon_{i^m k^m} \tag{17}$$

$$\Delta U_-^m = \Delta g_-^m - \epsilon_{i^m k^m}, \tag{18}$$

where

$$\begin{aligned} \Delta g_+^m &:= g\left(P_{i^m}^m \cup \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right) \\ &\quad - g\left(P_{i^m}^m; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right) \end{aligned} \tag{19}$$

$$\begin{aligned} \Delta g_-^m &:= g\left(P_{i^m}^m \setminus \{k^m\}; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right) \\ &\quad - g\left(P_{i^m}^m; b_{i^m}, w_{i^m}, \mathbf{A}, \mathbf{D}_{1,-i^m}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right). \end{aligned} \tag{20}$$

Probabilities of Meeting Outcomes

Because each (chooser, bidder) pair meets exactly once, the outcome of the meeting and their linkage in the initial state $\left[\mathbf{D}^0\right]_{i^m k^m}$ completely determine by their linkage in the terminal state $\left[\mathbf{D}^0\right]_{i^m k^m}$. Four cases are possible for the meeting (i^m, k^m):

1. $\left[\mathbf{D}^0\right]_{i^m k^m} = 0$ and $\left[\mathbf{D}^1\right]_{i^m k^m} = 0$
    - i^m does not add k^m to its portfolio of airline partners (maintains status quo).
    - i^m is better of with $P^m_{i^m}$ than $P^m_{i^m} \cup \{k^m\}$.
    - The probability is $\mathbb{P}\left(\Delta U^m_+ \leq 0\right) = \mathbb{P}\left(\epsilon_{i^m k^m} \leq -\Delta g^m_+\right)$.
2. $\left[\mathbf{D}^0\right]_{i^m k^m} = 0$ and $\left[\mathbf{D}^1\right]_{i^m k^m} = 1$
    - i^m adds k^m to its portfolio of airline partners.
    - i^m is better of with $P^m_{i^m} \cup \{k^m\}$ than $P^m_{i^m}$: $\Delta U^m_+ > 0$.
    - The probability is $\mathbb{P}\left(\Delta U^m_+ > 0\right) = \mathbb{P}\left(\epsilon_{i^m k^m} > -\Delta g^m_+\right)$.
3. $\left[\mathbf{D}^0\right]_{i^m k^m} = 1$ and $\left[\mathbf{D}^1\right]_{i^m k^m} = 0$
    - i^m removes k^m from its portfolio of airline partners.
    - i^m is better of with $P^m_{i^m} \setminus \{k^m\}$ than $P^m_{i^m}$: $\Delta U^m_- < 0$.
    - The probability is $\mathbb{P}\left(\Delta U^m_- \leq 0\right) = \mathbb{P}\left(\epsilon_{i^m k^m} \geq \Delta g^m_-\right)$.
4. $\left[\mathbf{D}^0\right]_{i^m k^m} = 1$ and $\left[\mathbf{D}^1\right]_{i^m k^m} = 1$
    - i^m does not remove k^m from its portfolio of airline partners (maintains status quo).
    - i^m is better of with $P^m_{i^m}$ than $P^m_{i^m} \setminus \{k^m\}$: $\Delta U^m_- \geq 0$.
    - The probability is $\mathbb{P}\left(\Delta U^m_- > 0\right) = \mathbb{P}\left(\epsilon_{i^m k^m} < \Delta g^m_-\right)$.

The probabilities are defined similarly for a meeting between a hotel chain and an airline, with $U(\cdot), g(\cdot), \Delta U, \Delta g$ replaced by $V(\cdot), h(\cdot), \Delta V, \Delta h$. With $V(\cdot), h(\cdot)$ as specified in (7), $\Delta V, \Delta h$ are defined similarly to $\Delta U, \Delta g$ as in the previous subsection.

If the m^{th} meeting is (j^m, k^m), where hotel chain j^m is the chooser rather than credit card issuer i^m, then the probabilities of the four cases are:

1. $\left[\mathbf{D}^0\right]_{j^m k^m} = 0$ and $\left[\mathbf{D}^1\right]_{j^m k^m} = 0$ has probability $\mathbb{P}\left(\Delta V_+^m \leq 0\right) = \mathbb{P}\left(\epsilon_{j^m k^m} \leq -h_+^m\right)$.

2. $\left[\mathbf{D}^0\right]_{j^m k^m} = 0$ and $\left[\mathbf{D}^1\right]_{j^m k^m} = 1$ has probability $\mathbb{P}\left(\Delta V_+^m > 0\right) = \mathbb{P}\left(\epsilon_{j^m k^m} > -\Delta h_+^m\right)$.

3. $\left[\mathbf{D}^0\right]_{j^m k^m} = 1$ and $\left[\mathbf{D}^1\right]_{j^m k^m} = 0$ has probability $\mathbb{P}\left(\Delta V_-^m \leq 0\right) = \mathbb{P}\left(\epsilon_{j^m k^m} \geq \Delta h_-^m\right)$.

4. $\left[\mathbf{D}^0\right]_{j^m k^m} = 1$ and $\left[\mathbf{D}^1\right]_{j^m k^m} = 1$ has probability $\mathbb{P}\left(\Delta V_-^m > 0\right) = \mathbb{P}\left(\epsilon_{j^m k^m} < \Delta h_-^m\right)$.

Conditional Likelihood Function

Let S^m denote the m meeting in S. S^m can be either (i^m, k^m) or (j^m, k^m). Under the i.i.d. assumption given by equation (2), the following conditional likelihood function describes transition from $\mathbf{D}^0$ to $\mathbf{D}^1$ conditional on S:

$$
\begin{aligned}
L\left(\theta; \mathbf{B}, \mathbf{W}, \mathbf{A}, \mathbf{D}^0, \mathbf{D}^1, S\right) = \prod_{m=1}^{M} & \mathbb{P}\left(\epsilon_{i^m k^m} \leq -\Delta g_+^m\right)^{\mathbb{1}\{S^m=(i^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{i^m k^m}=0,\left[\mathbf{D}^1\right]_{i^m k^m}=0\right\}} \\
& \times \mathbb{P}\left(\epsilon_{j^m k^m} \leq -\Delta h_+^m\right)^{\mathbb{1}\{S^m=(j^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{j^m k^m}=0,\left[\mathbf{D}^1\right]_{j^m k^m}=0\right\}} \\
& \times \mathbb{P}\left(\epsilon_{i^m k^m} > -\Delta g_+^m\right)^{\mathbb{1}\{S^m=(i^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{i^m k^m}=0,\left[\mathbf{D}^1\right]_{i^m k^m}=1\right\}} \\
& \times \mathbb{P}\left(\epsilon_{j^m k^m} > -\Delta h_+^m\right)^{\mathbb{1}\{S^m=(j^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{j^m k^m}=0,\left[\mathbf{D}^1\right]_{j^m k^m}=1\right\}} \\
& \times \mathbb{P}\left(\epsilon_{i^m k^m} \geq \Delta g_-^m\right)^{\mathbb{1}\{S^m=(i^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{i^m k^m}=1,\left[\mathbf{D}^1\right]_{i^m k^m}=0\right\}} \\
& \times \mathbb{P}\left(\epsilon_{j^m k^m} \geq \Delta h_-^m\right)^{\mathbb{1}\{S^m=(j^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{j^m k^m}=1,\left[\mathbf{D}^1\right]_{j^m k^m}=0\right\}} \\
& \times \mathbb{P}\left(\epsilon_{i^m k^m} < \Delta g_-^{t,m}\right)^{\mathbb{1}\{S^m=(i^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{i^m k^m}=1,\left[\mathbf{D}^1\right]_{i^m k^m}=1\right\}} \\
& \times \mathbb{P}\left(\epsilon_{j^m k^m} < \Delta h_-^m\right)^{\mathbb{1}\{S^m=(j^m,k^m)\}\times\mathbb{1}\left\{[\mathbf{D}^m]_{j^m k^m}=1,\left[\mathbf{D}^1\right]_{j^m k^m}=1\right\}}.
\end{aligned}
\tag{21}
$$

$\mathbf{B}$ denotes the collection of bids for all choosers. $\mathbf{W}$ denotes the collection of all potential conversion ratios[12].

7 Empirical Analysis

The dataset is consisted of 3 credit card issuers, 7 hotel chains, and 43 airlines, as listed in table 2. The dataset contains annual observations of their partnerships, which is the network of their loyalty programs, from November 2014 to November 2018. At firm-level, the dataset contains quarterly observations of key

[12] $\mathbf{B}$ contains vector of bids b_i for all credit card issuers $i \in \mathbb{I}$ and b_j for all hotel chains $j \in \mathbb{J}$. $\mathbf{W}$ contains vector of potential conversion ratios w_i for all $i \in \mathbb{I}$ and w_j for all $j \in \mathbb{J}$. As described in the "chooser" subsection of section 5, b_i, b_j, w_i, w_j belong to $\mathbb{R}_{++}^{|\mathbb{K}|}$, where $\mathbb{K}$ is the set of airlines.

performance indicators (KPIs) and characteristics of their loyalty programs from Q1 2014 to Q4 2018. For airlines, the dataset also contains geographic locations of hubs and the number of loyalty points (often called "miles") necessary to redeem for reward flights to various geographic locations. All dataset entries were manually obtained from publicly available information, such as 10Q, 10K, financial statement (for non-U.S. firms), earnings call transcript, and official corporate announcement. The online appendix documents details of the data collection procedure.

Sections 5 and 6 describes the transition of the network from an initial state $\mathbf{D}^0$ to a terminal state $\mathbf{D}^1$. The empirical analysis puts together two transitions. Specifically, $\mathbf{D}^0$ denotes the November 2014 network, $\mathbf{D}^1$ denotes the November 2016 network, and $\mathbf{D}^2$ denotes the November 2018 network. The conditional likelihood function describes the probability for the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$ conditional on a sequence of meetings and then from $\mathbf{D}^1$ to $\mathbf{D}^2$ conditional on another sequence of meetings. The two sequences of meetings are assumed to be independent from each other. All model parameters are identical for the two transitions.

Firm-level datasets were aggregated to match the two transitions. *Period 1* denotes the eight quarters from Q1 2015 to Q4 2016, and it corresponds to the transition from $\mathbf{D}^0$ to $\mathbf{D}^1$. *Period 2* denotes the eight quarters from Q1 2017 to Q4 2018, and it corresponds to the transition from $\mathbf{D}^1$ to $\mathbf{D}^2$. Additionally, *Period 0* denotes the four quarters from Q1 2014 to Q4 2015. All firm-level observations were averaged over the quarters to construct period-level observations.

Preliminary Analysis

Figure 6 in the appendix reports KPIs and the number of partners (both airlines and hotel chains) of credit card issuers for periods 0,1, and 2. JPMC consistently has the largest *Sales*, which is amount of purchases made via credit card products. On the other hand, AMEX consistently has the smallest *Delinquency*, which is the percentage of credit card loans that are past due for 30 days or more. AMEX also consistently has the smallest *Writeoff*, which is the percentage of credit card loans that are written off[13]. It suggests AMEX's customers are associated with the smallest credit risk. Moreover, AMEX consistently has the largest *#Partners*, which is the number of partners (hotel chains and airlines), followed by CITI and then JPMC.

Figure 7 in the appendix reports KPIs and the number of airline partners of hotel chains. MAR consis-

[13]Loans that are long past due are written off and sold to collection agencies.

tently has the largest *Revenue*, which is total operating revenue, indicating it has the largest overall market size. MAR also has the largest #Partners, is the number of airline partners. On the other hand, HYT consistently has the largest *RevPAR* (revenue per available room), although it has the smallest *Hotels* (number of hotels properties). Large RevPAR indicates that on average, the hotel chain is high-end because customers are willing to pay high price to stay at its properties. *OCP* (occupancy rate) is the percentage of occupied rooms relative to total available rooms. OCP varies across hotel chains, but they are not dramatically different. Hotel chains generally have more transfer partners than credit card issuers. Note that SPG disappears in period 2 because it was acquired by MAR.

Table 3 in the appendix reports summary statistics of KPIs of airlines. On average, *PaxRev* (passenger revenue) fell in period 1 and then increased in period 2. PaxRev captures overall market size of passenger transportation services, excluding freight services. On average, *RPK* (revenue passenger kilometers) increased steadily over the periods. RPK is total flight distance of sold seats in kilometers, and it is a measure of quantity demanded for passenger flight services. On average, *ASK* (available seat kilometers) also increased steadily. ASK is total total flight distance of sold and sellable seats, and it is a measure of supply or capacity of passenger flight services. For all periods, the distribution of every KPI exhibits a long right tail, indicating there is a small number of airlines with large market size. Large standard deviation relative to the mean and wide $[P_{25}, P_{75}]$ and $[P_{10}, P_{90}]$ percentile ranges indicate there is substantial variation in the market size of airlines. Note that there are only 41 airlines in period 2 because AB and SA were omitted. AB went bankrupt in 2017. In 2017, SA shrank in size significantly due to financial hardships.

OLS Fit of Bids

The dataset does not include observations of bids. Instead, the dataset contains conversion ratios for (credit card issuer, airline) or (hotel chain, airline) pairs with partnership and real values of loyalty points. Applying equation (8) yields observed bids for pairs with partnership. This cannot be done for pairs without partnership because their (potential) conversion ratios are not observed. The unobserved bids, for pairs without partnership, are constructed by fitting linear regression models. Afterwards, potential conversion ratios are constructed for all pairs without partnership using equation (8). Note that the dataset contains observations of partnership, conversion ratio, and values of loyalty points for periods 0,1, and 2.

The unobserved bids for (credit card issuer, airline) pairs are constructed by (1) computing the OLS fit of observed bid b_{ik} onto period dummy variables and cubic polynomials of all KPIs of credit card issuer i and airline k, and then (2) computing predictions by plugging in periods and values of KPIs of the unobserved pairs. The unobserved bids for (hotel chain, airline) pairs are constructed similarly with a separate OLS fit. Table 4 in the appendix reports the OLS fits.

Table 5 in the appendix compares summary statistics of observed bids and constructed bids. For both OLS fits, the constructed bids are smoothed out, as the standard deviation is smaller than the observed bids. For (hotel chain, airline) pairs, constructed bids have larger minimum and smaller maximum than the observed bids. Overall, the summary statistics of observed and constructed bids are similar. The constructed bids and potential conversion ratios are treated as data when estimating the parameters of the conditional likelihood function.

Main Result

Table 6 reports the estimated parameters of credit card issuer's objective function. Table 7 reports the estimated parameters of hotel chain's objective function. They are obtained from 11,866 MCMC draws, which remain after removing the first half of the draws (after the "burn-in" process). The precise estimation procedure is documented in the online appendix.

Table 6 reports the main estimation result. **Mean** and **Median** report the average and the 50^{th} percentile of the 11,866 MCMC draws, respectively. $[\mathbf{P_{2.5}}, \mathbf{P_{97.5}}]$ reports the 2.5 to 97.5 percentile range of the draws, which has 95 percentage coverage. Parameters β_1 and β_2 capture how network interactions affect the choice of airline partners for credit card issuers. β_3, β_4, and β_5 capture how the marginal contribution of an airline to the credit card issuer's loyalty program, in addition to existing airline partners, affect the choice of airline partners. It is not meaningful to interpret β_6 and β_7 because they are coefficients associated with the control variables, which are the number of airline partners and bid-weighted average KPIs of airline partners. Fixed effect specific to each credit card issuer, α_i, is differenced away (see Marginal Contribution to Objective Function subsection in section 6).

Consider the m^{th} meeting between credit card issuer i and airline k, which is not i's partner at the beginning of the meeting. Hold everything fixed and also consider that the m^{th} meeting is between i and a different airline k', which is also not i's partner at the beginning of the meeting. Suppose k and k' are identical except for exactly one of the following:

1. k is a partner of another credit card issuer, k' is not.

2. k is a partner of i's hotel chain partner(s), k' is not.

3. Adding k as a partner lowers the overall cost of points for reward flights (lower *Mileage*) offered by i's loyalty program, adding k' instead lowers it by less.

4. Adding k as a partner increases the diverseness of flight routes (larger *Routes*) for reward flights offered by i's loyalty program, adding k' instead increases it by less.

5. Adding k as a partner increases the geographic diverseness ($GeoHub$) of airline hubs associated with i's loyalty program, adding k' instead does not change it.

Similar to equation (19), define

$$\Delta g_{+}^{m}\left(k;\beta\right) := g\left(P_i^m \cup \{k\}\,; b_i, w_i, \mathbf{A}, \mathbf{D}_{1,-i}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right) - g\left(P_i^m, b_i, w_i, \mathbf{A}, \mathbf{D}_{1,-i}^{0,m}, \mathbf{D}_2^{0,m}, \mathbf{D}_3, \mathbf{D}_4, \beta\right), \tag{22}$$

where $g(\cdot)$ is given equation (3). The odds ratio of i adding k as a partner at the m^{th} meeting relative to adding k' is

$$\frac{\exp\left(\Delta g_{+}^{m}\left(k;\beta\right)\right)}{\exp\left(\Delta g_{+}^{m}\left(k';\beta\right)\right)}. \tag{23}$$

If the odds ratio is larger (smaller) than 1, then i is more (less) likely to add k as a partner than $k^{'}$, holding the state of the network and characteristics of k and k' constant.

The estimation result suggests:

1. β_1 is positive with 90% statistical significance. A credit card issuer is more likely to add an airline partner that is a partner of another credit card issuer than otherwise.

2. β_2 is positive with 95% statistical significance. A credit card issuer is more likely to add an airline partner that is a partner of its hotel chain partner(s) than otherwise.

3. β_3 is negative with 95% statistical significance. A credit card issuer is more likely to add an airline partner that better complements its portfolio of airline partners so that reward flights offered by its loyalty program are more cost-effective.

4. β_4 is negative with 95% statistical significance. A credit card issuer is less likely to add an airline partner that better complements the portfolio of airlines so that reward flights offered by its loyalty program cover more diverse flight routes.

5. β_5 is not different from 0 with 90% or higher statistical significance. There is no evidence that a credit card issuer is more or less likely to add an airline partner that complements the geographic diverseness of airline hubs associated with its loyalty program.

The result for β_1 and β_2 indicates that network relationships affect the choice of an airline partner, above and beyond actual gains of the credit card issuer's loyalty program and the standalone characteristics of the airline. The result for β_3 and β_4 suggests that when choosing an airline partner, credit card issuers value complementarity to the existing portfolio of airline partners, above and beyond standalone characteristics of the airline. Unlike other covariates, adding an airline partner more or less linearly contributes to the bid-weighted average of KPIs. In other words, adding an airline partner with KPIs larger (smaller) than the average of the current portfolio increases (decreases) the average. Thus, changes in bid-weighted average KPIs resulting from adding an airline partner account for non-complementary, standalone characteristics of the airline.

The result reported in table 7 suggests that for hotel chains, network relationships similarly affect the choice of an airline partner, above and beyond actual gains to its loyalty program and the standalone characteristics of the airline. However, estimation result for γ_3, γ_4, and γ_5 suggests hotel chains have different preferences for complementarity when choosing an airline partner. A plausible reason is that hotel chains have significantly more airline partners than credit card issuers.

8 Concluding Remark

Using novel dataset and a sequential network formation model, this article shows that the choice of partners by firms may depend on network-based relationships, above and beyond their standalone characteristics. In my knowledge, this is the first empirical study of network formation that involve distinct classes of agents - credit card issuers, hotel chains, and airlines - with a structure that certain types of links are impossible. Credit cards and their rewards have important consequences to consumer welfare, for both credit card users and cash users, and the transfer of wealth between the two groups, as Schuh et al. (2010) suggests. Regretfully, this article does not include a discussion of counterfactual consequences to consumer welfare associated with various configurations of the network of loyalty programs. It is due to the lack of observations on consumer demand for credit card products and transactions using loyalty points. Future research may address it and provide insight on the welfare consequences of the network of loyalty programs.

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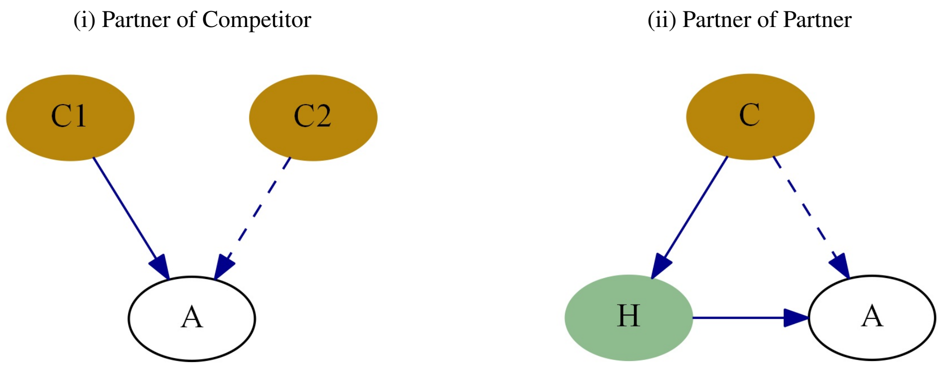
Appendix

Figure 1: Examples of Network Relationships



(i) C1 and C2 are credit card issuers, and A is an airline. C2 could be more or likely to choose A as a partner because A is a partner of its competitor C1. (ii) C is a credit card issuer, H is a hotel chain, and A is an airline. C could be more or less likely to choose A as a partner because A is a partner of it partner hotel chain H.

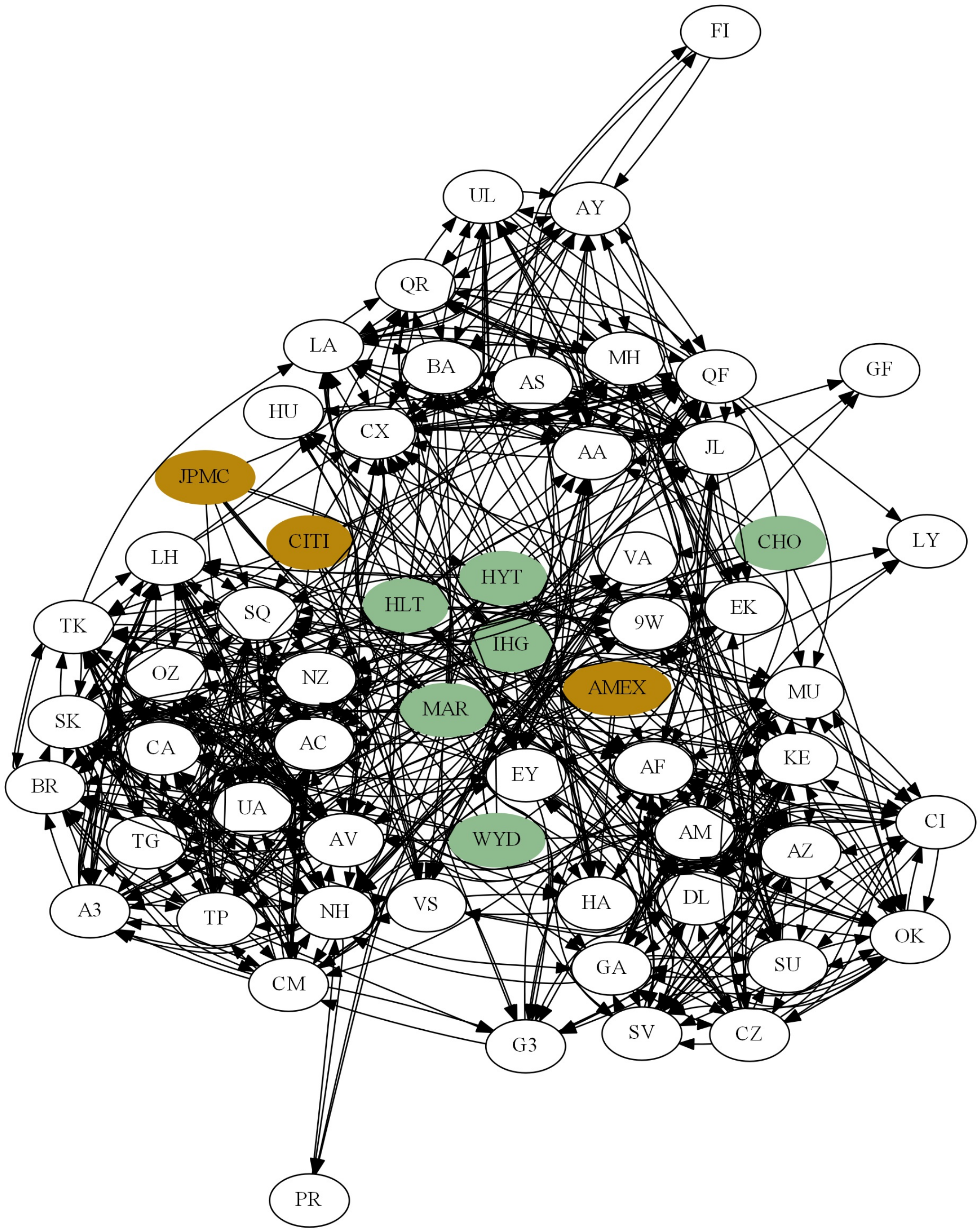
Table 1: History of Credit Card Issuers' Partner

Time	AMEX	JPMC	CITI
2013 Q2		Adds VS (Jun.)	
2013 Q3			
2013 Q4	Adds EK (Oct.)		
2014 Q1			
2014 Q2		Adds SQ (May)	Adds CX (Jul.), BR (Jul.), EY (Jul.), GA (Jul.), QR (Jul.), SQ (Jul.), TG (Jul.), MH (Aug.), AF (Aug.)
2014 Q3			
2014 Q4		Loses KE (Nov.)	
2015 Q1	Loses F9 (Jan.)	Adds KE (Jan.)	Adds VS (Jan.), QF (Feb.)
2015 Q2			
2015 Q3			
2015 Q4	Updates BA[14] (Oct.)		
2016 Q1			
2016 Q2	Adds EY (Apr.)	Adds AF (May.)	
2016 Q3	Adds MAR (Sep.); Loses SPG (Sep.)[15]		
2016 Q4			Adds B6 (Oct.)
2017 Q1			Removes VX[16] (Jan.)
2017 Q2	Updates BA[17] (Jul.)		Adds 9W (Apr.)
2017 Q3			Adds TK (Aug.)
2017 Q4	Removes VX (Nov.)	Adds EI (Nov.), IB (Nov.)	Adds AV (Nov.); Loses HLT (Dec.)
2018 Q1	Updates HLT[18] (Jan.)		
2018 Q2			
2018 Q3	Adds EI (Aug.)	Loses KE (Aug.); Adds B6 (Aug.)	Updates B6[19] (Sep.)
2018 Q4	Adds AV (Nov.)		
2019 Q1	Adds QF (May)		
2019 Q2			
2019 Q3		Adds EK (Aug.)	Loses GA (Aug.)

This table reports the history of changes to transfer partnerships possessed by AMEX, JPMC and CITI, from the second quarter of 2013 to the third quarter of 2019. Abbreviations of firms' names are listed in table 2.

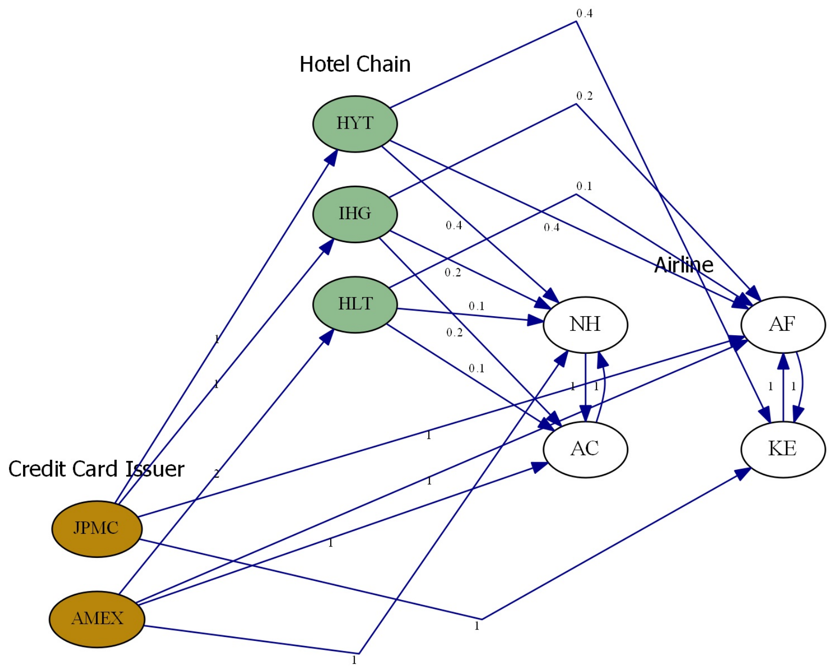
[14] Conversion ratio was updated from 1 to 0.8.
[15] MAR acquired SPG. 3:1 transfer between them. SPG was removed from the network of loyalty programs and treated as a part of MAR.
[16] AS acquired VX in December 2016.
[17] Conversion ratio was updated from 0.8 to 1.
[18] Conversion ratio was updated from 1.5 to 2.
[19] Conversion ratio was updated from 0.8 to 1.

Figure 2: Network of Loyalty Programs (Nov. 2018)



This figure illustrates the network of loyalty programs observed in November 2018. The golden, teal, and white nodes are the loyalty programs of credit card issuers, hotel chains, and airlines, respectively. See table 2 in the appendix for a dictionary of node names. A directed link from a node to another indicates that points may be transferred from the source node to the target node. Nodes are positioned close to each other if they are strongly connected via direct and indirect links. The cluster of airlines on the bottom-left is Star Alliance, the cluster on the bottom-right is SkyTeam, and the cluster on the top is Oneworld. AMEX is positioned close to SkyTeam. On the other hand, JPMC and CITI are positioned close to Star Alliance and Oneworld.

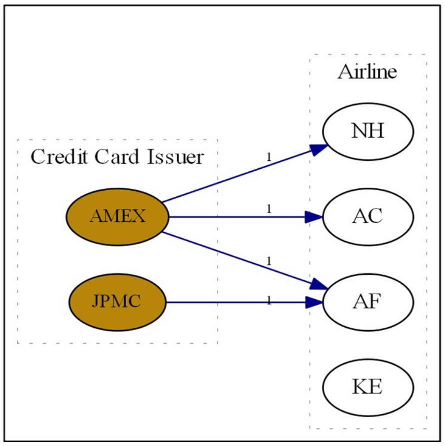
Figure 3: Abridged Network of Loyalty Programs (Nov. 2018)



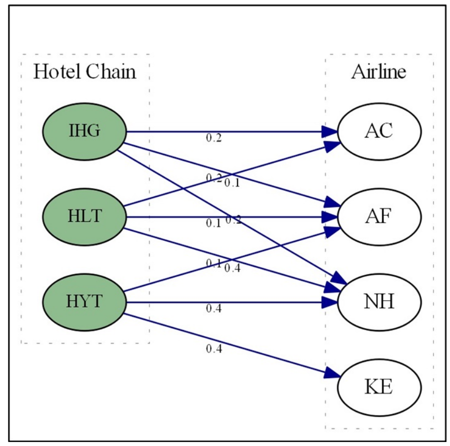
This figure is an abridged version of figure 2, with only a subset of the nodes. The golden, teal, and white nodes are the loyalty programs of credit card issuers, hotel chains, and airlines, respectively. A directed link from a node to another indicates that points may be transferred from the source node to the target node. Except for the links between airlines, each link weights indicates the conversion ratio per 1 point in the source node. A link between airlines indicate that points in the source node can be used to redeem for flights offered by the target node. All links between airline nodes are assigned a weight of 1.

Figure 4: Splitting into Subnetworks

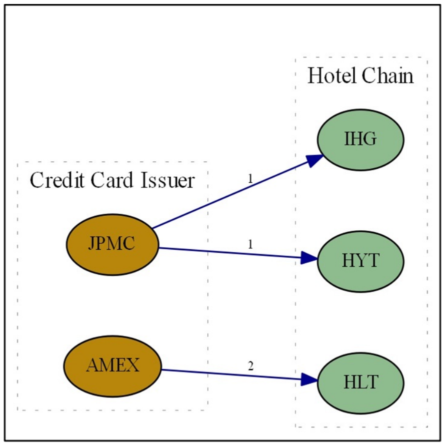
(i) Subnetwork $\mathbf{D}_1$



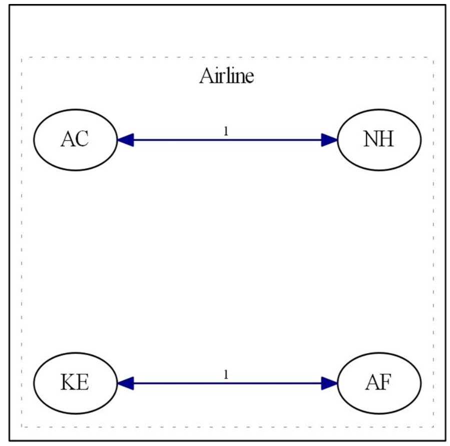
(ii) Subnetwork $\mathbf{D}_2$



(iii) Subnetwork $\mathbf{D}_3$



(iv) Subnetwork $\mathbf{D}_4$



The network in figure 3 is split into four subnetworks. Subnetwork 1 (upper left) describes linkages from credit card issuers to airlines. Subnetwork 2 (upper right) describes linkages from hotel chains to airlines. Subnetwork 3 (lower left) describes linkages from credit card issuers to hotel chains. Subnetwork 4 (lower right) describes linkages between airlines.

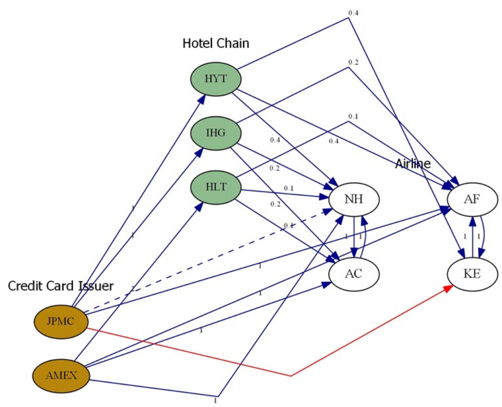
Table 2: Firm Names and Abbreviations

ID	Name	ID	Name
AMEX	American Express Company	FI	Icelandair
CITI	Citibank	GA	Garuda Indonesia
JPMC	J.P. Morgan Chase Bank	GF	Gulf Air
CHO	Choice Hotels International	HA	Hawaiian Airlines
HLT	Hilton Hotels and Resorts	HU	Hainan Airlines
HYT	Hyatt Hotels	JL	Japan Airlines
IHG	Intercontinental Hotels Group	KE	Korean Air Lines
MAR	Marriott Hotels and Resorts	LA	LATAM Airlines
SPG	Starwood Hotels and Resorts	LH	Lufthansa
WYD	Wyndham Hotels and Resorts	LY	El Al Israel Airlines
9W	Jet Airways	MH	Malaysia Airlines
A3	Aegean Airlines	MU	China Eastern Airlines
AA	American Airlines	NH	All Nippon Airways
AB	Air Berlin	NZ	Air New Zealand
AC	Air Canada	OK	Czech Airlines
AF	Air France/KLM	OZ	Asiana Airlines
AM	Aeromexico	PR	Philippine Airlines
AS	Alaska Airlines	QF	Qantas Airways
AV	Avianca	QR	Qatar Airways
AY	Finnair	SA	South African Airways
AZ	Alitalia	SK	Scandinavian Airlines
BA	British Airways	SQ	Singapore Airlines
BR	EVA Air	SU	Aeroflot
CA	Air China	SV	Saudia
CI	China Airlines	TG	Thai Airways
CM	Copa Airlines	TK	Turkish Airlines
CX	Cathay Pacific Airways	TP	TAP Air Portugal
CZ	China Southern Airlines	UA	United Airlines
DL	Delta Air Lines	UL	SriLankan Airlines
EK	Emirates	VA	Virgin Australia
EY	Etihad Airways	VS	Virgin Atlantic

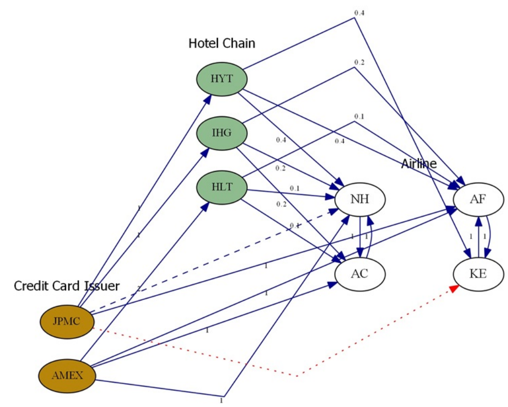
This table provides a dictionary for abbreviations of firm names. For airlines, the abbreviation is the IATA (International Air Transportation Association) code.

Figure 5: Dependence on Sequence of Meetings

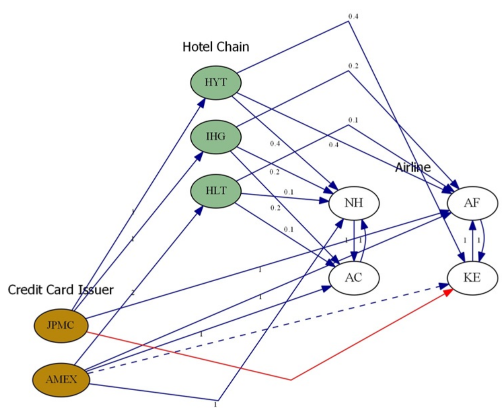
(i) (JPMC, NH) occurs before (JPMC, KE)



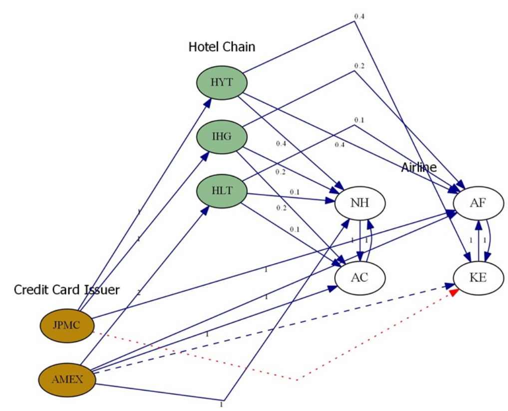
(ii) (JPMC, NH) occurs after (JPMC, KE)



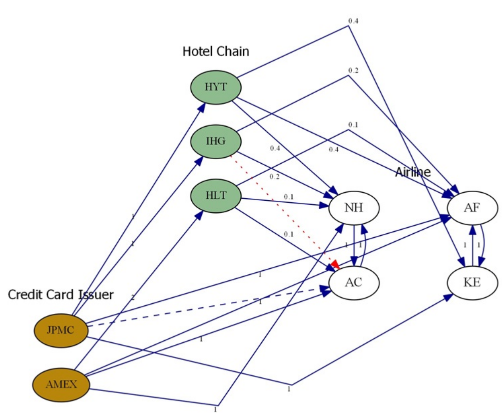
(iii) (AMEX, KE) occurs before (JPMC, KE)



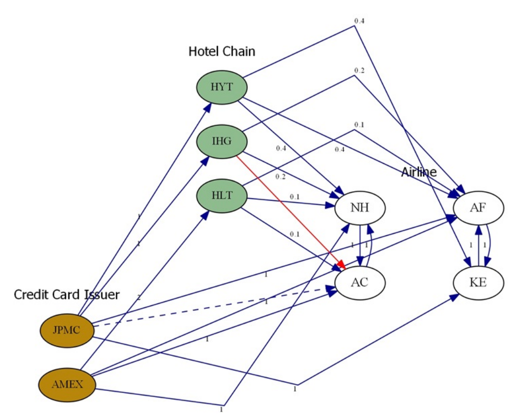
(iv) (AMEX, KE) occurs after (JPMC, KE)



(v) (JPMC, AC) occurs before (IHG, AC)

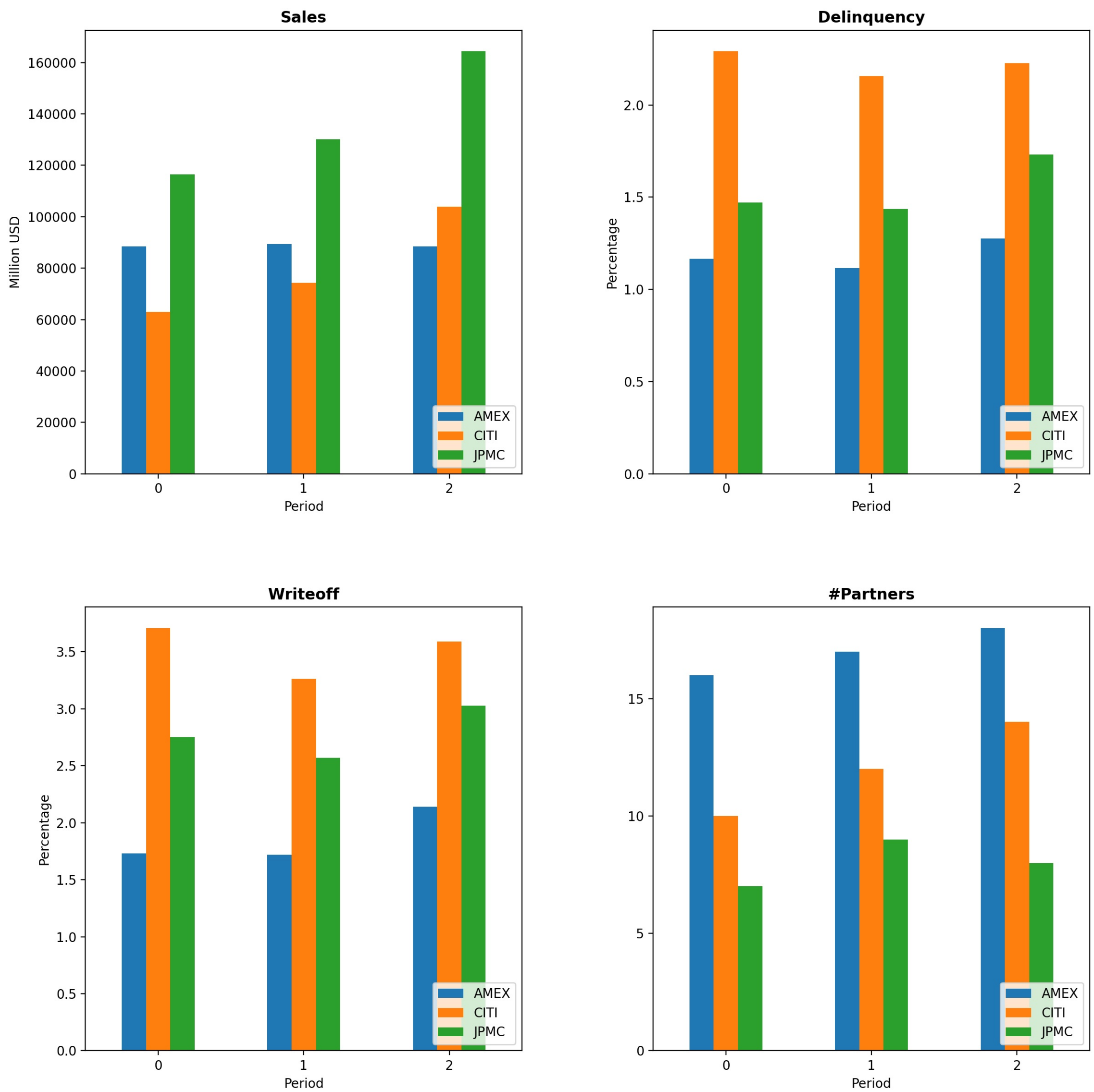


(vi) (JPMC, AC) occurs after (IHG, AC)



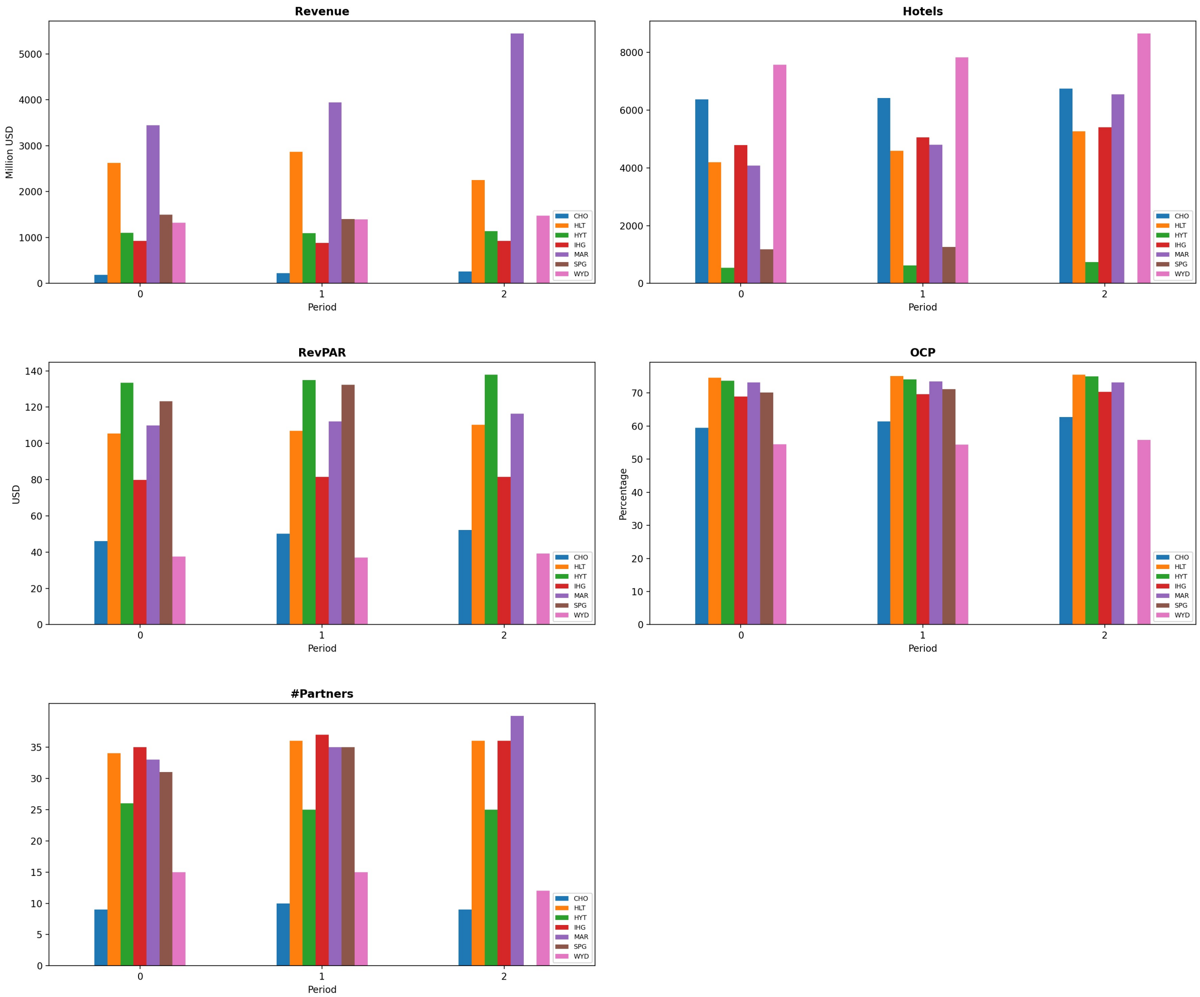
Sub-figures (i) and (ii) illustrate how the order of a credit card issuer's own meetings may affect the current state of the network it faces. In (i), JPMC meets NH before the link from JPMC to KE was removed. In (ii), JPMC meets NH after the link from JPMC to KE was removed. (iii) and (iv) illustrate how the order of another credit card issuer's meetings may affect the current state of the network. In (iii), AMEX meets KE before the link from JPMC to KE was removed. In (iv), AMEX meets KE after the link from JPMC to KE was removed. (v) and (vi) illustrate how the order of a hotel chain's meetings may affect the current state of the network. In (v), JPMC meets AC before the link from IHG to AC was added. In (vi), JPMC meets AC after the link from IHG to AC was added.

Figure 6: KPIs of Credit Card Issuers



This figure reports summary statistics of KPIs (key performance indicators) and the number of partners for credit card issuers. Periods 0, 1, and 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. For each period, the reported numbers are averages over the quarters. The reported numbers include only the measures associated with U.S. consumers. Sales is total amount of purchases made using the firm's credit card products. Delinquency is the percentage of outstanding loans that are past due for at least 30 days. Writeoff is the share of net-writeoff in outstanding loans. #Partners is the number of the partners, including both hotel chains and airlines.

Figure 7: KPIs of Hotel Chains



This figure reports summary statistics of KPIs (key performance indicators) and the number of airline partners for hotel chains. Periods 0, 1, 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. For each period, the reported numbers are averages over the quarters. Revenue is the total operating revenue of the hotel chain, including revenue from rooms and franchise fees. Hotels is the number of worldwide hotel properties owned and leased by the hotel chain. RevPar (revenue per available room) is room revenue, from both owned and leased properties, divided by the number of available rooms. OCP (occupancy rate) is the number rooms sold divided by the number of available rooms. #Partners is the number of airline partners.

Table 3: Summary Statistics of Airline KPIs

Period	Obs	Variable	Mean	SD	10^{th}	25^{th}	50^{th}	75^{th}	90^{th}
0	43	PaxRev (Million USD)	2,390	2,562	515	731	1,189	2,800	7,182
		RPK (Million KM)	23,447	22,680	6,194	7,931	14,299	27,887	56,824
		ASK (Milllion KM)	29,206	27,286	7,838	10,200	20,742	34,642	71,323
1	43	PaxRev (Million USD)	2,173	2,329	499	667	1,078	2,593	5,623
		RPK (Million KM)	25,614	23,933	6,133	9,250	15,414	30,750	63,342
		ASK (Million KM)	31,764	28,989	8,073	11,635	21,065	38,733	74,668
2	41	PaxRev (Million USD)	2,562	2,682	600	765	1,250	2,959	6,882
		RPK (Million KM)	29,747	26,276	8,176	9,707	19,750	35,765	69,758
		ASK (Million KM)	36,342	31,715	9,914	12,168	24,759	44,388	84,039

This table reports summary statistics of key performance indicators (KPIs) of airlines. Periods 0, 1, and 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. Obs reports the number of airlines. For each period, the reported numbers are averages over the quarters. PaxRev (passenger revenue) is revenue from scheduled and chartered flights. RPK (revenue passenger KM)is total flight distance of sold seats in kilometers. ASK (available seat KM) is total flight distance of sold and sellable passenger seats in kilometers.

Table 4: OLS Fit of Bids

Variable	Credit Card Issuer	Hotel Chain
Sales	0.0246 (0.022)	
Sales^2	-0.0002 (0.000)	
Sales^3	7.391e-07 (6.17e-07)	
Delinquency	0.0040 (0.346)	
Delinquency^2	-0.0028 (0.046)	
Delinquency^3	-0.0286 (0.048)	
Writeoff	0.0153 (0.481)	
Writeoff^2	0.0303 (0.259)	
Writeoff^3	-0.0032 (0.038)	
Revenue		-0.0600 (0.099)
Revenue^2		0.0292 (0.032)
Revenue^3		-0.0035 (0.003)
Hotels		3.023e-05 (4.88e-05)
Hotels^2		7.822e-09 (1.3e-08)
Hotels^3		-5.341e-13 (9.34e-13)
RevPAR		0.0359 (0.028)
RevPAR^2		-0.0003 (0.000)
RevPAR^3		9.534e-07 (7.52e-07)
OCP		0.0550 (0.048)
OCP^2		-0.0020 (0.002)
OCP^3		1.477e-05 (1.36e-05)
RPK	0.0167 (0.055)	-0.0165* (0.009)
RPK^2	-0.0005 (0.001)	0.0005** (0.000)
RPK^3	5.759e-06 (9.6e-06)	-3.483e-06** (1.47e-06)
ASK	0.0110 (0.043)	0.0153** (0.008)
ASK^2	-0.0002 (0.001)	-0.0004** (0.000)
ASK^3	1.738e-07 (4.8e-06)	2.016e-06** (7.6e-07)
PaxRev	-0.2057** (0.084)	-0.0018 (0.020)
PaxRev^2	0.0650** (0.024)	-0.0014 (0.005)
PaxRev^3	-0.0054** (0.002)	0.0001 (0.000)
Period 0 Dummy	0.0292 (0.130)	-0.0128 (0.009)
Period 1 Dummy	-0.0061 (0.130)	-0.0157** (0.005)
Period 2 Dummy	-0.0201 (0.141)	0.0305** (0.010)
Adj. R-squared	0.150	0.129
No. observations	79	459

This table reports OLS fit of the linear regression models for bids. The first column lists the covariates included in the linear regression models. For definitions, see the Preliminary Analysis subsection of section 7. The second column reports parameter estimates associated with the bids received by credit card issuers. The third column reports parameter estimates associated with the bids received by hotel chains. Standard errors are reported in parentheses. ** indicates statistical significance at 0.05 level. * indicates statistical significance at 0.1 level.

Table 5: Summary Statistics of Bids

	Mean	SD	Min	10th	25th	50th	75th	90th	Max
Observed Bids for Credit Card Issuers [79 pairs]	0.933	0.113	0.640	0.752	0.900	0.963	1.012	1.045	1.082
Constructed Bids for Credit Card Issuers [381 pairs]	0.902	0.073	0.653	0.807	0.851	0.904	0.950	0.996	1.114
Observed Bids for Hotel Chains [459 pairs]	0.262	0.066	0.071	0.182	0.217	0.259	0.304	0.345	0.564
Constructed Bids for Hotel Chains [848 pairs]	0.258	0.026	0.212	0.229	0.239	0.254	0.275	0.295	0.354

This table reports observed bids (constructed from observed variables) and fitted bids. The row named Observed Bids for Credit Card Issuers reports summary statistics of the bids received by credit card issuers from airlines, computed using 79 (Credit Card Issuer, Airline) pairs with partnership. Constructed Bids for Credit Card Issuers reports summary statistics of the predicted bids for 381 (Credit Card Issuer, Airline) pairs without partnership. Observed Bids for Hotel Chains reports summary statistics of the bids received by Hotel Chains from airlines, computed using 459 (Hotel Chain, Airline) pairs with partnership. Constructed Bids for Hotel Chains reports summary statistics of the predicted bids for 848 (Hotel Chain, Airline) pairs without partnership.

Table 6: Estimation Result of Credit Card Issuer's Objective Function

Parameter	Variable	Mean	Median	$[\mathbf{P_{2.5}}, \mathbf{P_{97.5}}]$
β_1	*Competitor*	0.5100*	0.5605	[-0.0392,1.1371]
β_2	*Transitivity*	0.7848**	0.7398	[0.1476,1.4663]
β_3	*Mileage*	-0.9683**	-0.9792	[-1.3119,-0.6260]
β_4	*Route*	-0.1999**	-0.2097	[-0.3370,-0.6021]
β_5	*GeoHub*	-0.1431	-0.1294	[-0.5185,0.3309]
β_6	#Partners	-0.5381	-0.4253	[-1.8700,0.7010]
$\beta_{7,1}$	PaxRev (part of *Performance*)	-0.2233	-0.3045	[-0.6589,0.3340]
$\beta_{7,2}$	RPK (part of *Performance*)	-0.0834	0.0025	[-0.6042,0.2937]
$\beta_{7,3}$	ASK (part of *Performance*)	0.2304	0.1218	[-0.3017,0.9099]

This table reports estimation result for the parameters of the credit card issuer's objective function. **Mean** reports the mean of the MCMC draws. **Median** reports the median of the draws. $[\mathbf{P_{2.5}}, \mathbf{P_{97.5}}]$ reports the 2.5 - 97.5 percentile range of the draws. ** indicates that the 2.5 - 97.5 percentile range does not contain zero. * indicates that the 5 - 95 percentile range does not contain zero.

Table 7: Estimation of Hotel Chain's Objective Function

Parameter	Variable	Mean	Median	$[\boldsymbol{P_{2.5}}, \boldsymbol{P_{97.5}}]$
γ_1	*Competitor*	2.1022**	2.1444	[1.5285,2.5603]
γ_2	*Transitivity*	1.3304**	1.3328	[0.9639,1.7671]
γ_3	*Mileage*	-0.0386**	-0.0374	[-0.0817,-0.0018]
γ_4	*Routes*	0.0638*	0.0668	[-0.0070,0.1164]
γ_5	*GeoHub*	-1.0087**	-1.0148	[-1.2977,-0.6702]
γ_6	#Partners	-1.6273**	-1.7723	[-2.1376,-0.5865]
$\gamma_{7,1}$	PaxRev (part of *Performance*)	0.8543**	0.8323	[0.5138,1.1762]
$\gamma_{7,2}$	RPK (part of *Performance)*	-0.2898*	-0.2957	[-0.5436,0.0214]
$\gamma_{7,3}$	ASK (part of *Performance)*	0.0472	0.0243	[-0.1943,0.3308]

This table reports estimation result for the parameters of the hotel chain's objective function. **Mean** reports the mean of the MCMC draws. **Median** reports the median of the draws. $[\mathbf{P_{2.5}}, \mathbf{P_{97.5}}]$ reports the 2.5 - 97.5 percentile range of the draws. ** indicates that the 2.5 - 97.5 percentile range does not contain zero. * indicates that the 5 - 95 percentile range does not contain zero.