COMPARISON OF THE MOST POPULAR METHODS FOR RECONSTRUCTION MRI IMAGES

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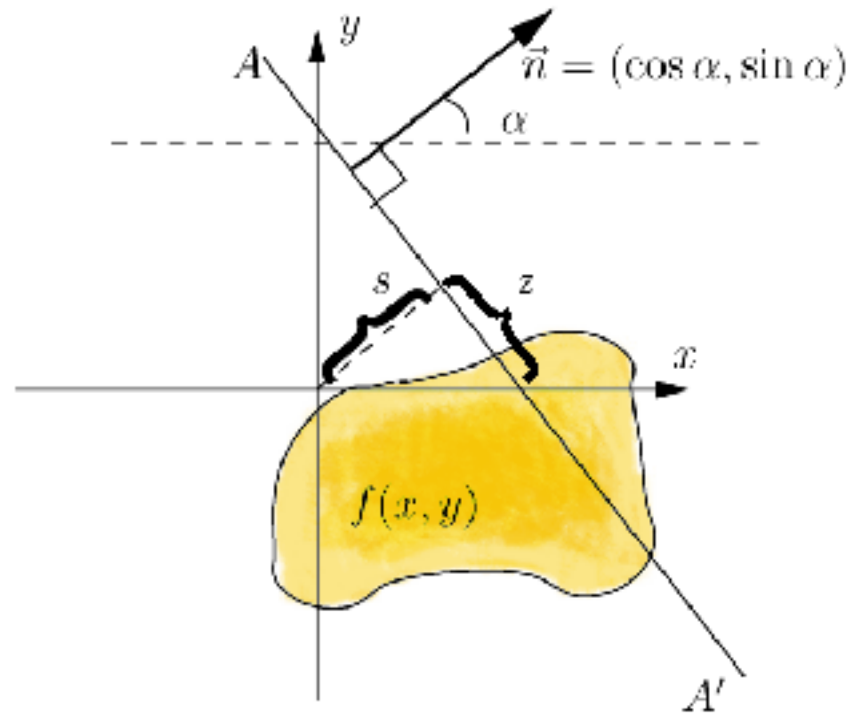
Direct Radon Transform

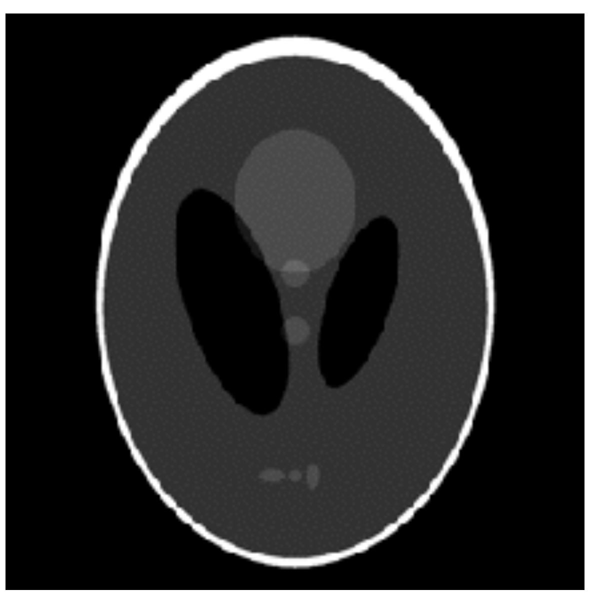


$$\hat{R}f(AA') = \int_{AA'} f(\mathbf{x})d\mathbf{x}$$

$f(x, y)$ - absorption function

α - observation angle

image



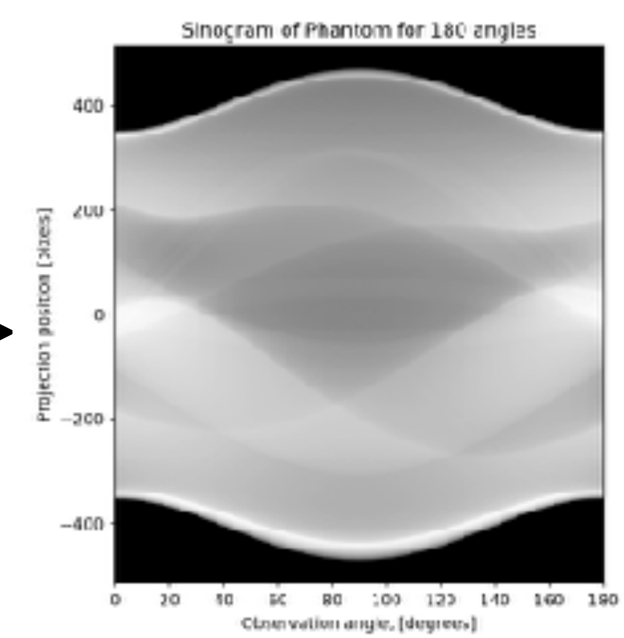
line AA' can be parametrized by it's distance s to the origin

and the angle α, that AA' makes with y axes

$$\hat{R}f(\alpha, s) = \int_{-\infty}^{\infty} f(x(z), y(z))dz =$$

$$= \int_{-\infty}^{\infty} f(z\sin\alpha + s\cos\alpha, -z\cos\alpha + s\sin\alpha)dz.$$

sinogram

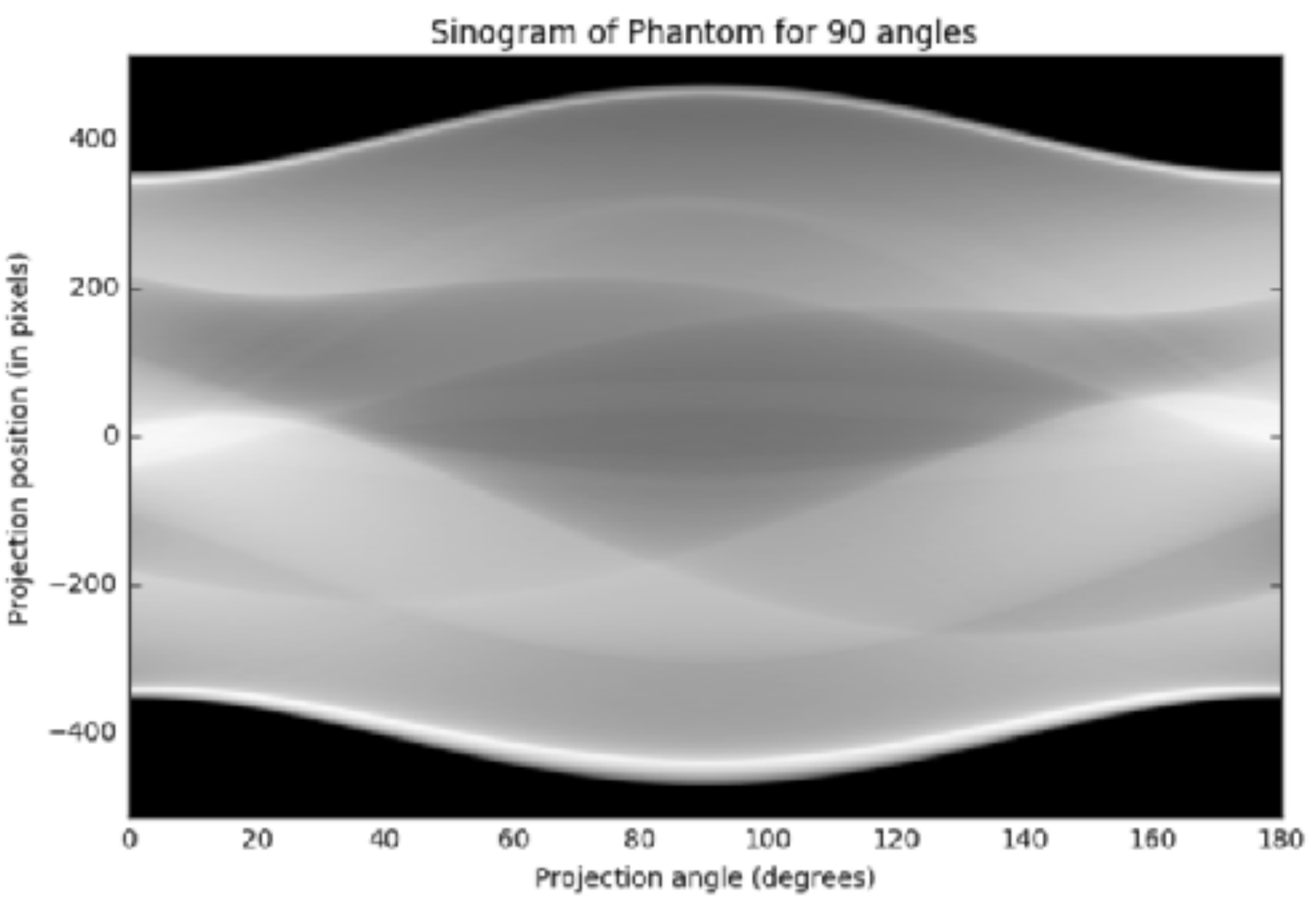


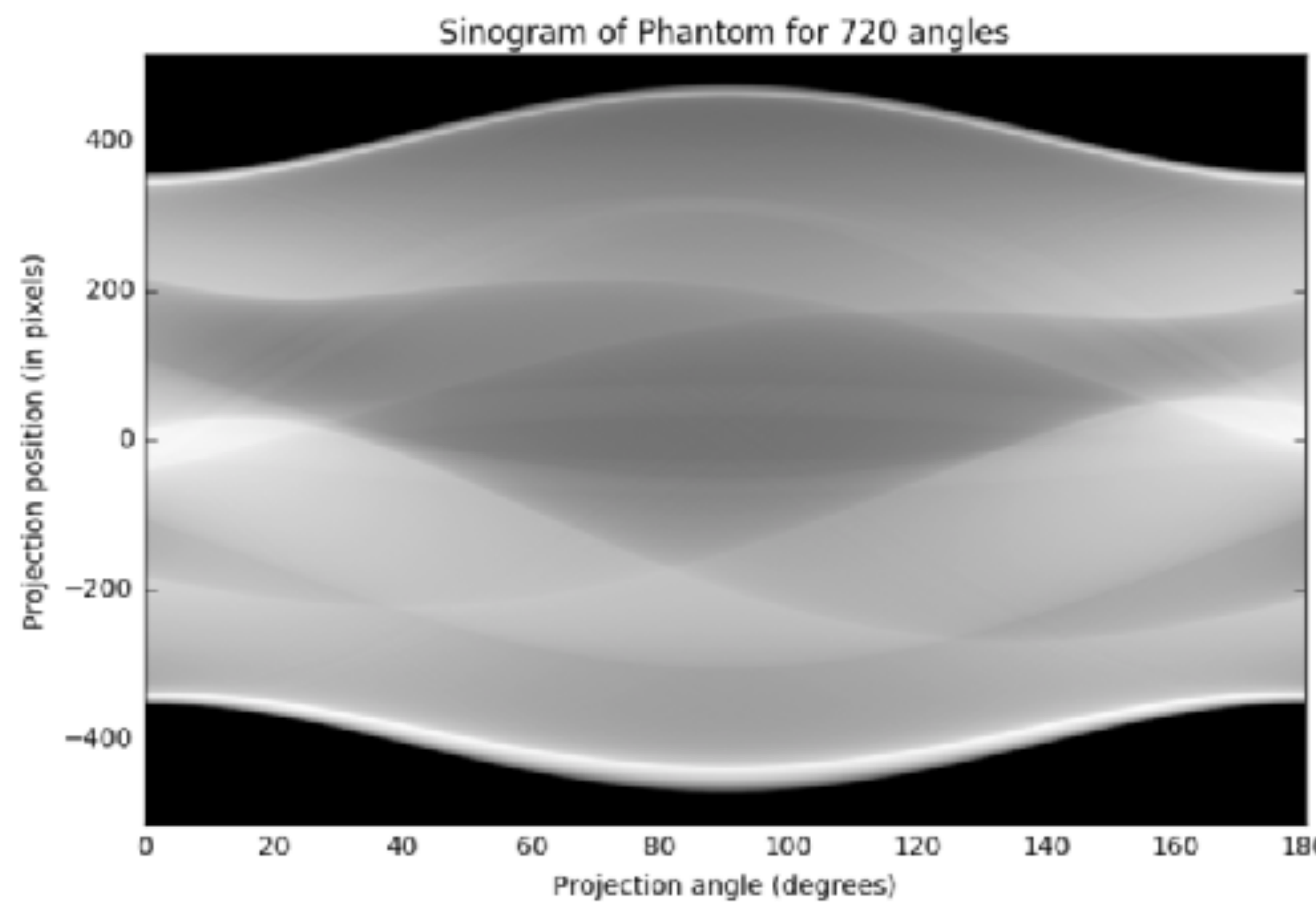
Direct Radon Transform - sinograms

Sinograms with different numbers of observation angles:

PHANTOM

90 ANGLES | **720 ANGLES**



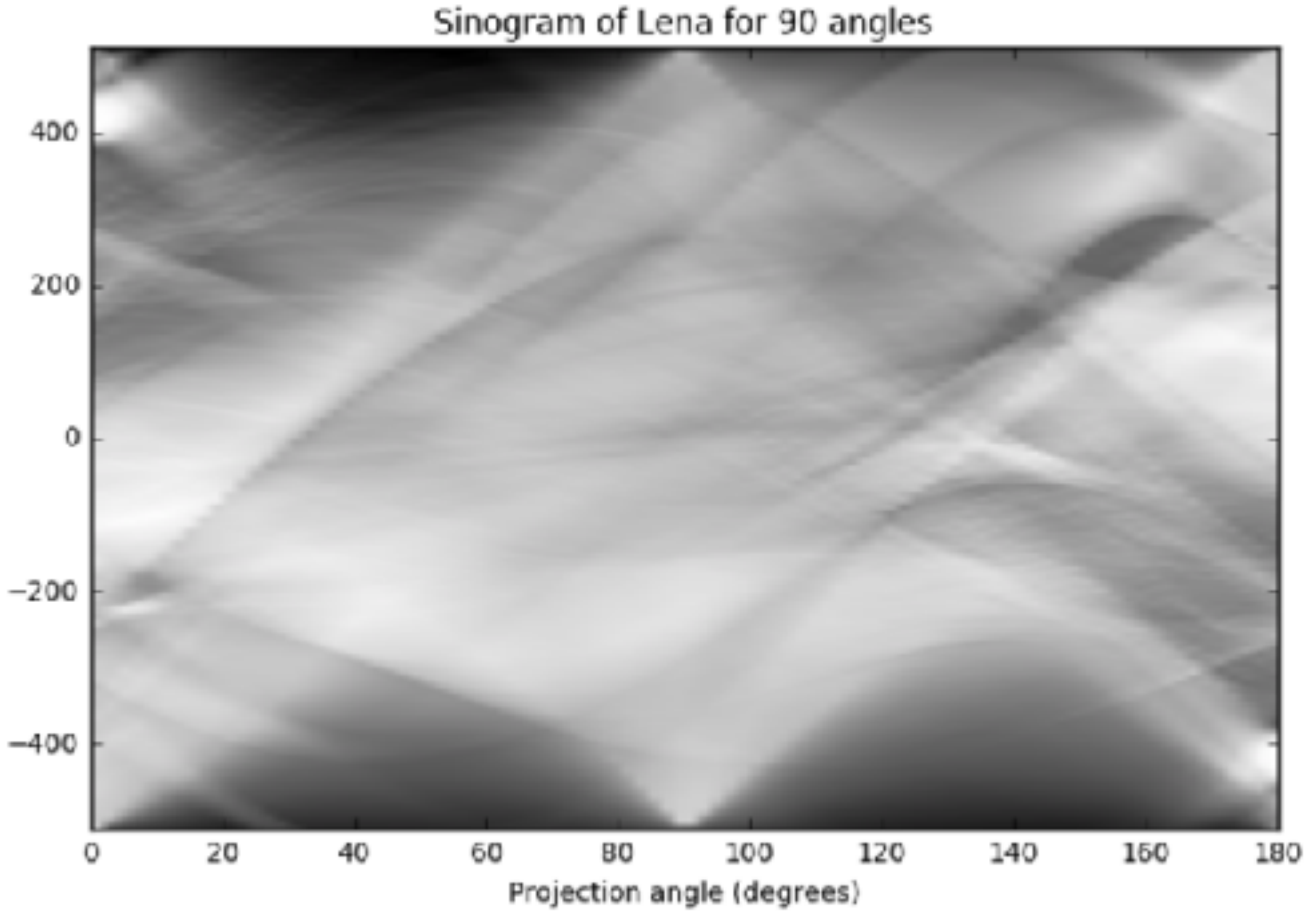


There is no difference visually — effect of piecewise-constant structure: it's not necessary to do lots observations to make quite accurate sinogram

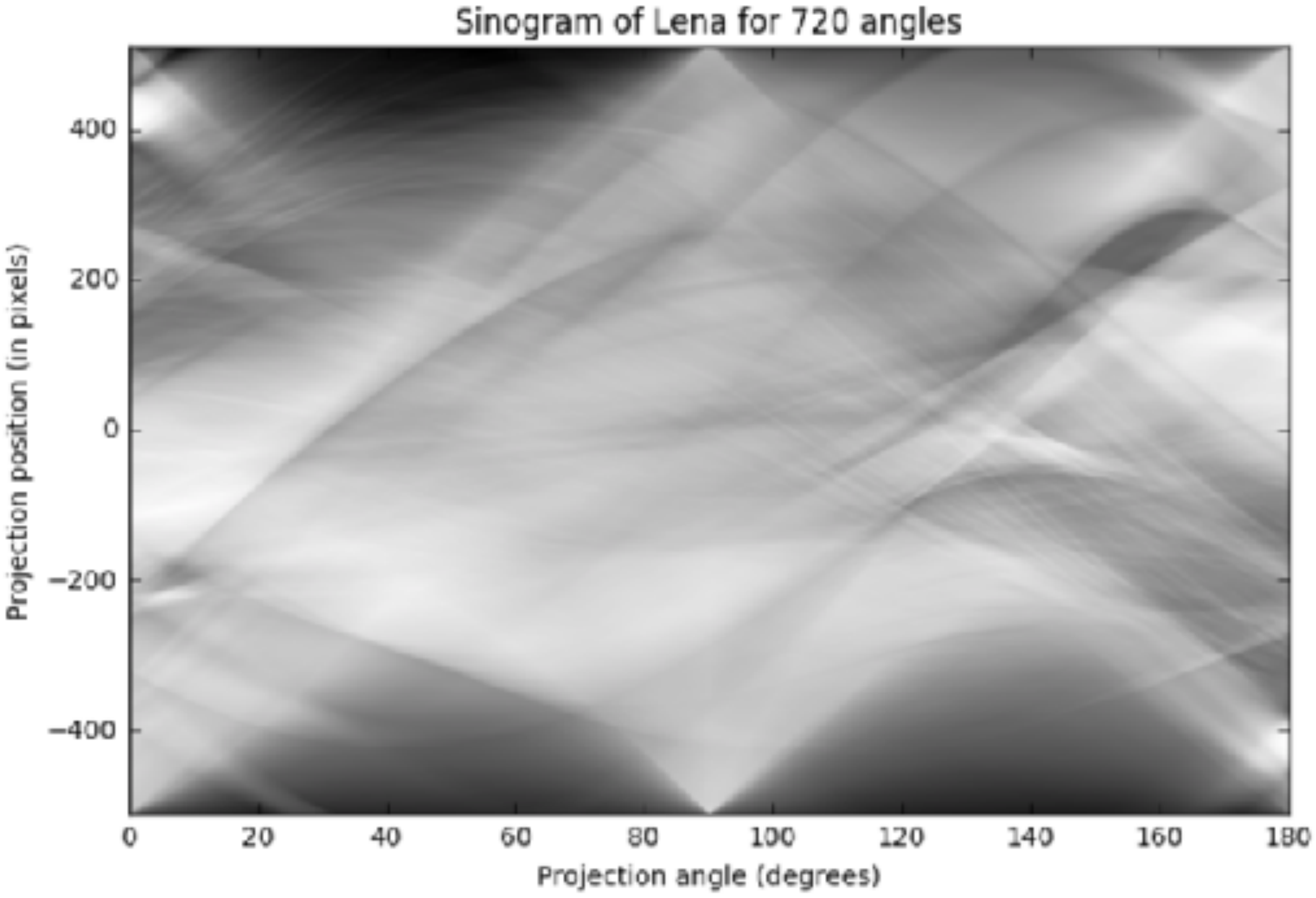
Direct Radon Transform - sinograms

LENA

90 ANGLES



720 ANGLES

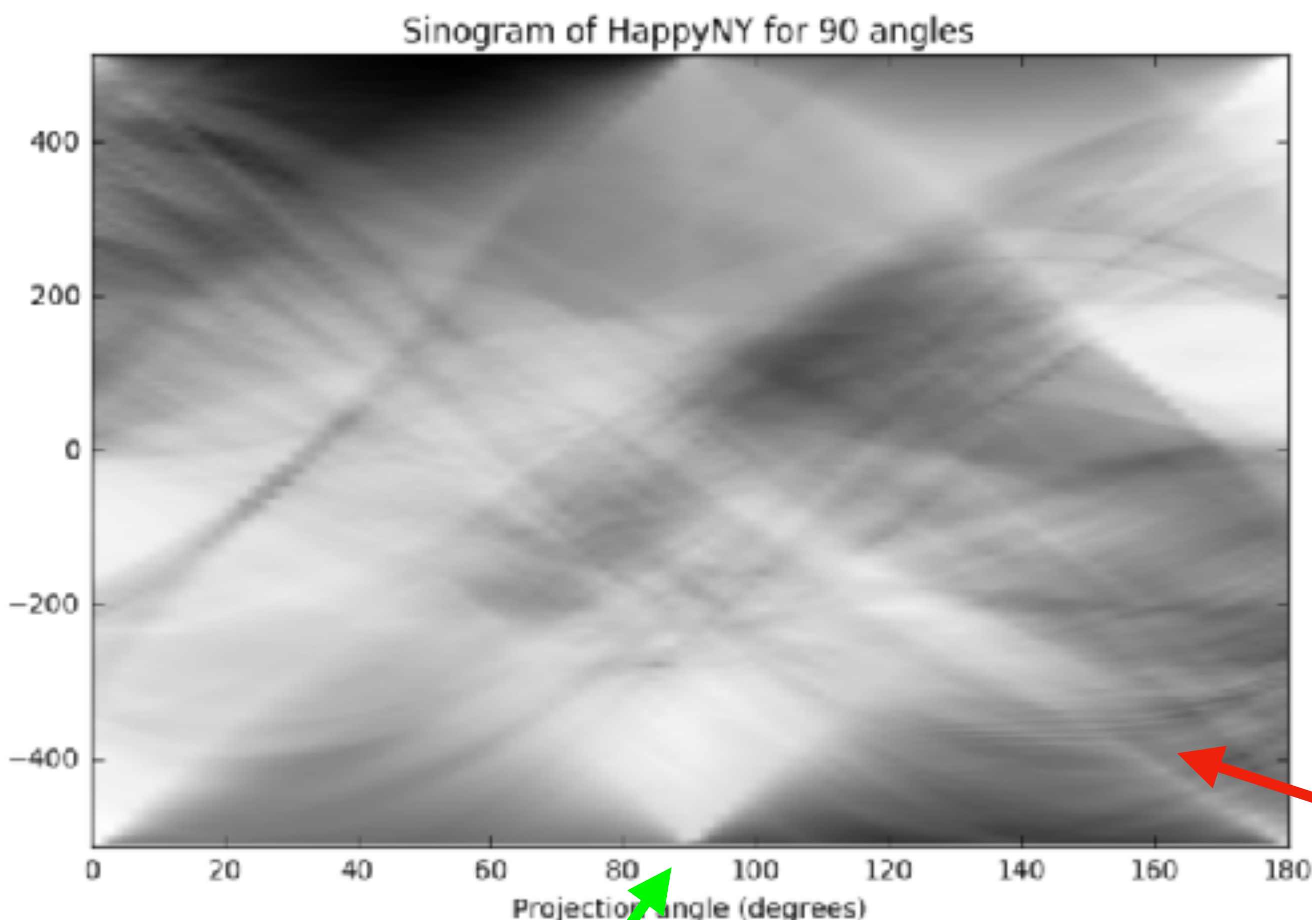


For not piecewise-constant objects we have even visually recognized difference in sinograms

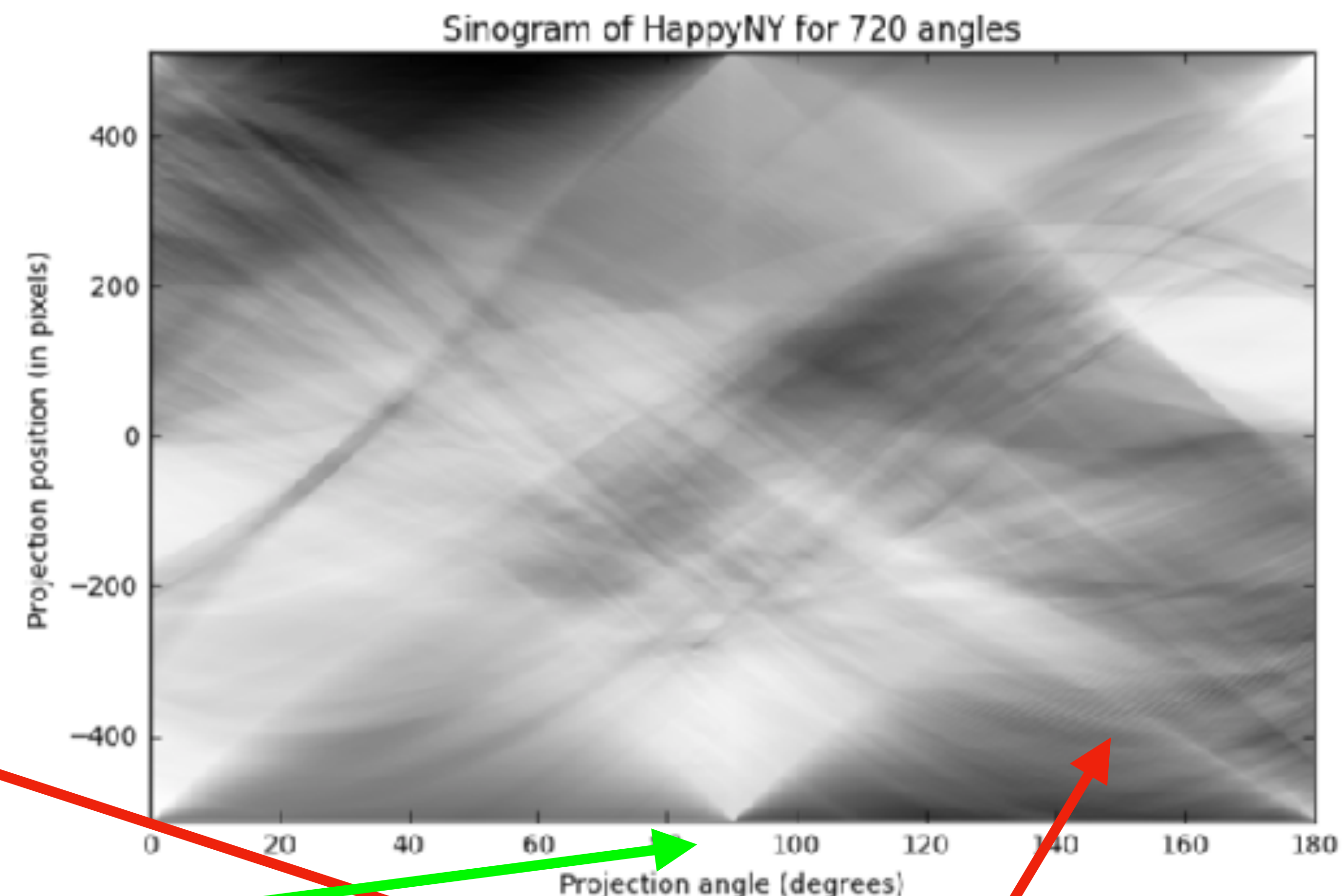
Direct Radon Transform - sinograms

HappyNY

90 ANGLES



720 ANGLES



Blurred peak

Different directions of lines

There are also visual differences

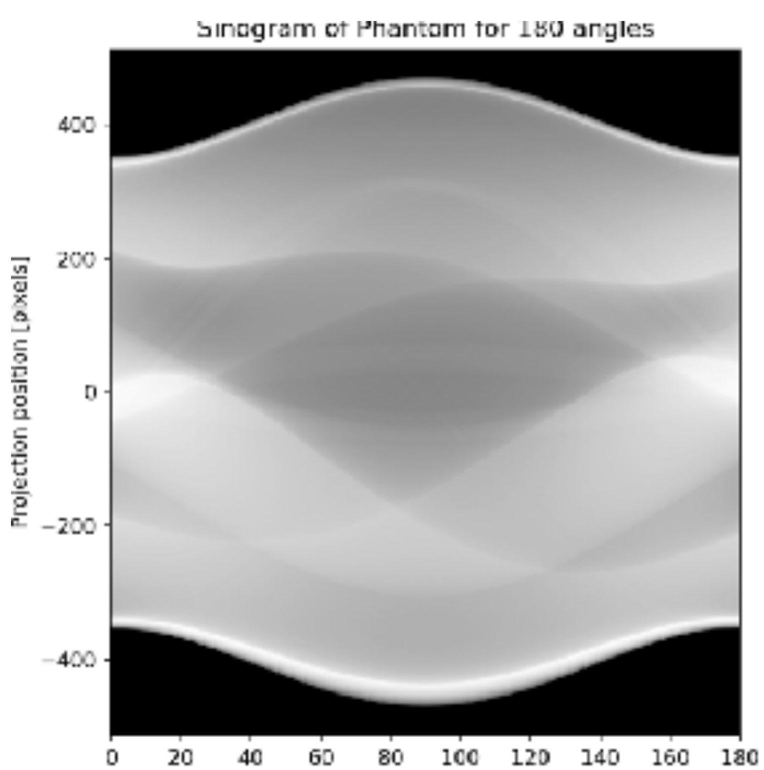
Direct Radon Transform - sinograms

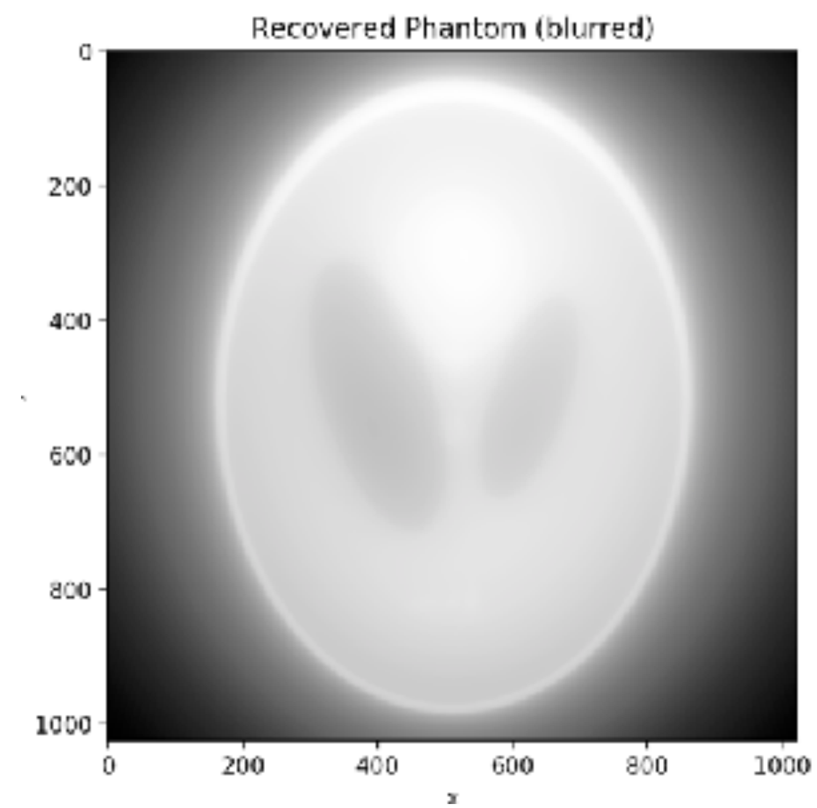
SUMMARY:

MORE OBSERVATION ANGLES — MORE ACCURACY!

But what about reconstruction?

Inverse Radon Transform - image reconstruction





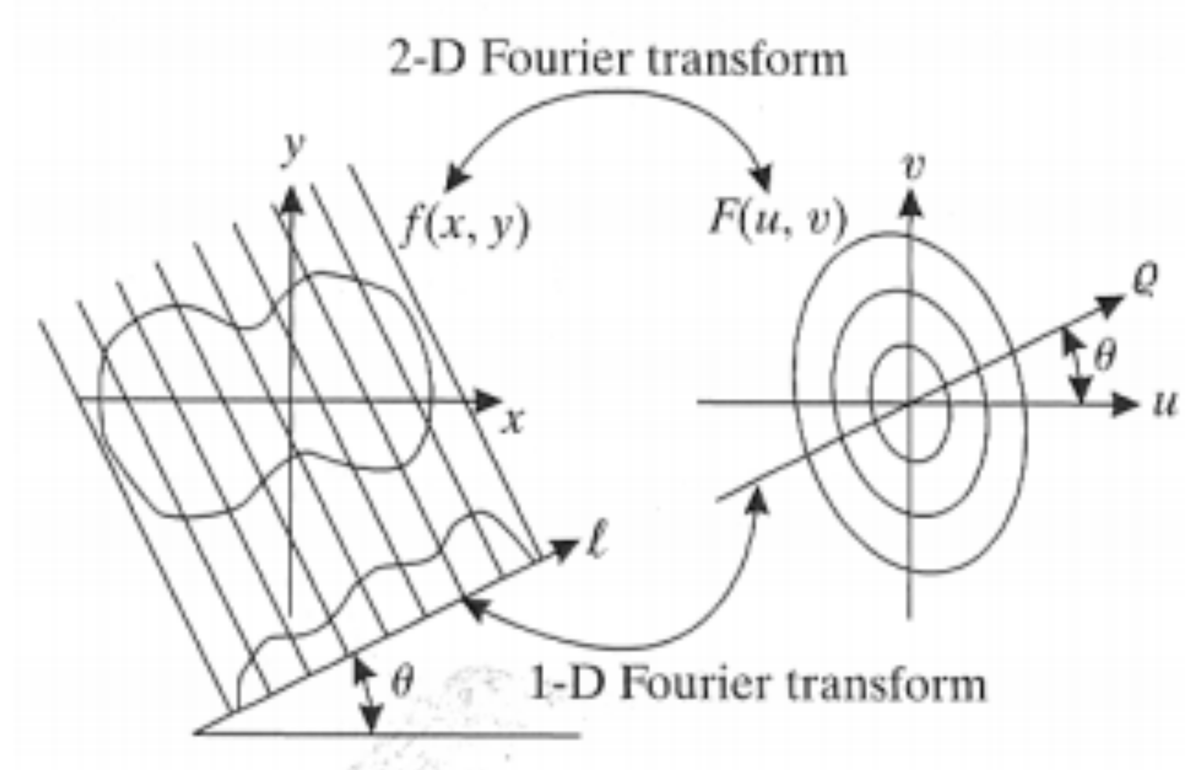
There are several ways to reconstruct image from set of projections; we implemented the most famous of them:

- Reconstruction, based on **Fourier Slice Theorem**
- Reconstruction using **Dual Radon Transform** (back projection)

Approach I: Inverse Radon Transform via Fourier Slice Theorem

Given a real-valued function f defined on the plane, then

$$\hat{f}(\lambda\left(\begin{smallmatrix}\cos\varphi\\ \sin\varphi\end{smallmatrix}\right)) = \frac{1}{\sqrt{2\pi}}\widehat{R_f(\varphi,\cdot)}(\lambda)$$



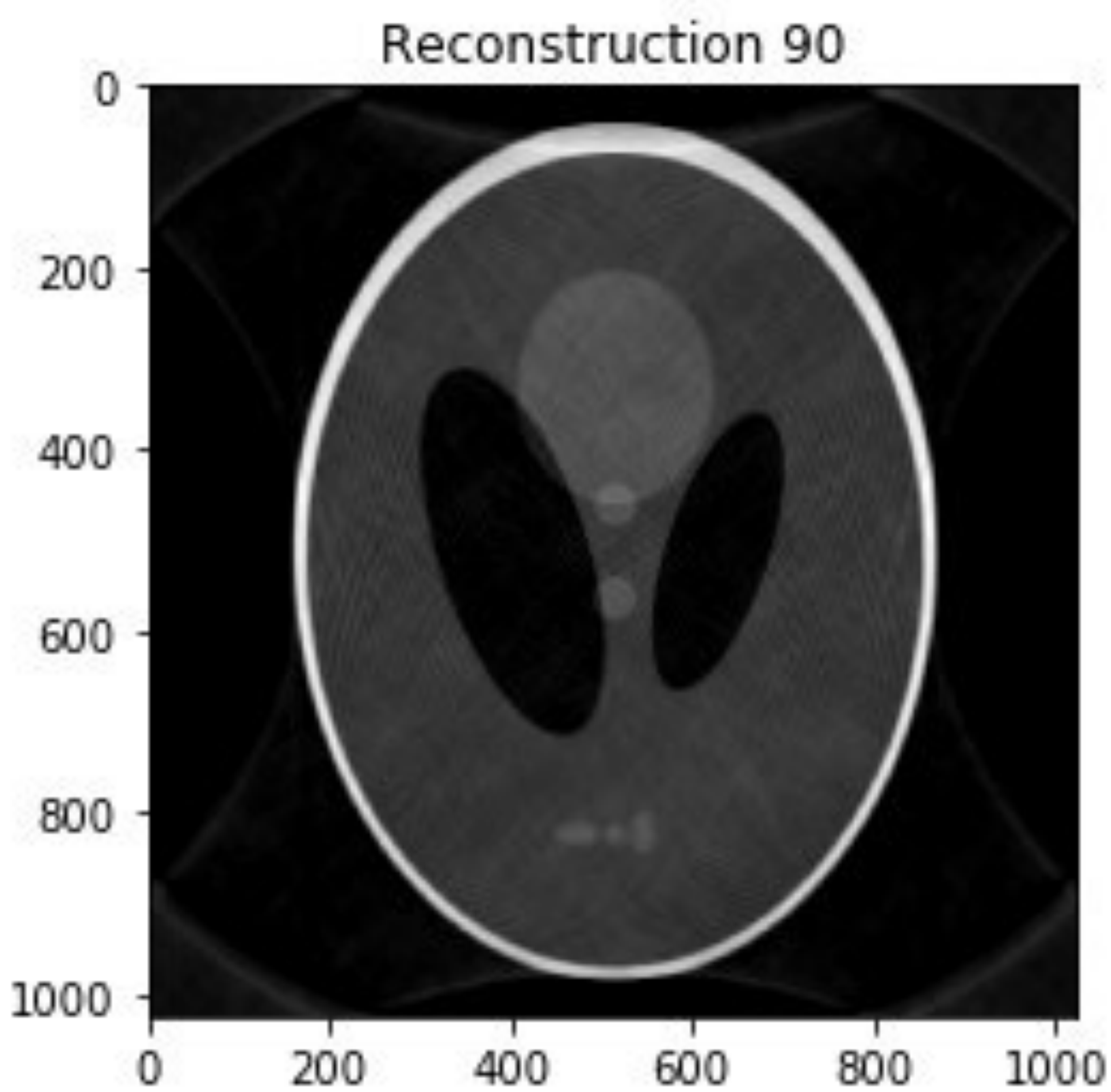
Sketch of the algorithm:

1. Take one dimensional Fourier transforms of the given Radon transform $\hat{R}f(\varphi,\cdot)$, for a (hopefully large) number of angles φ.
2. Take the inverse two dimensional Fourier transform of the above result.

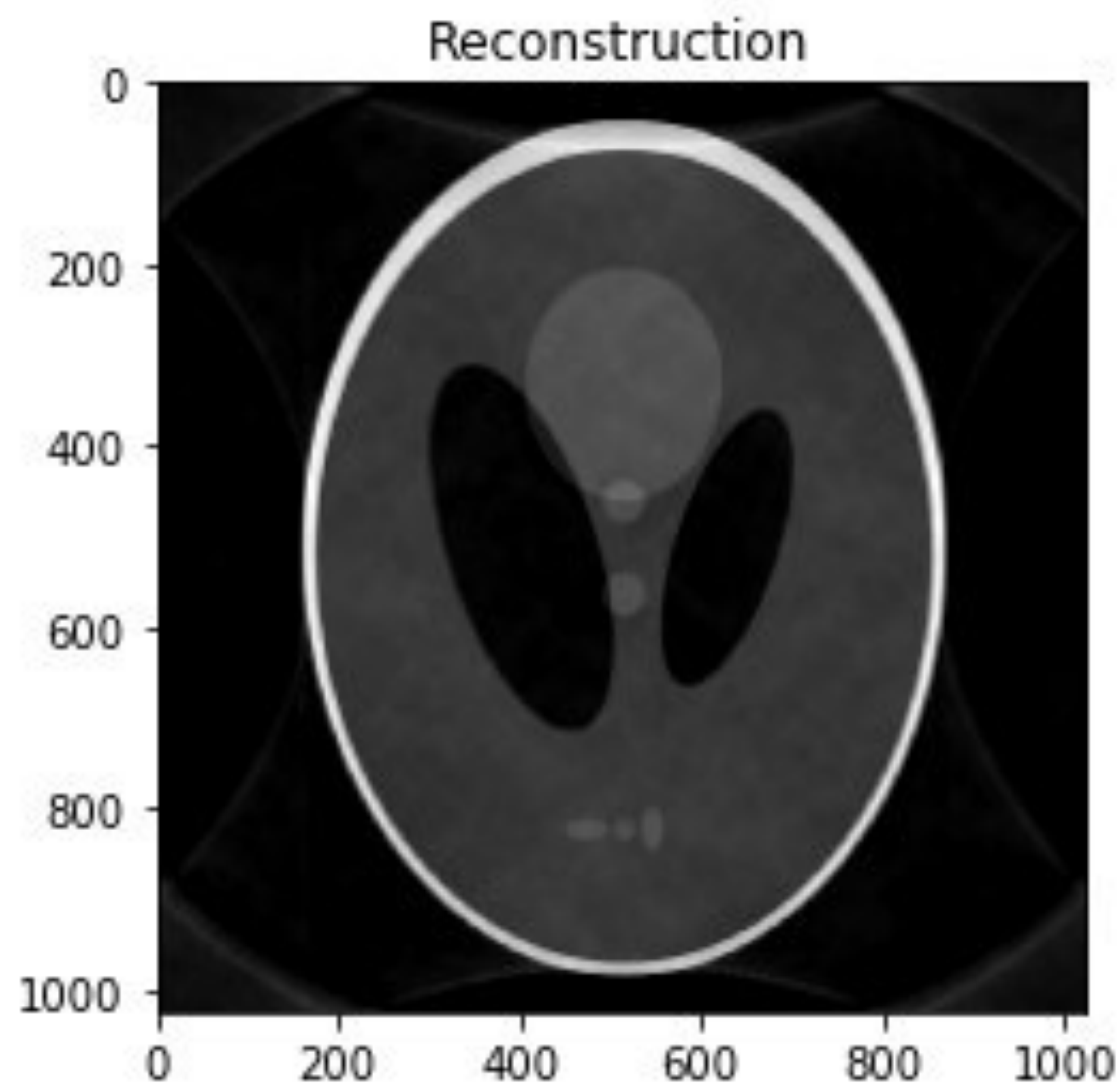
Inverse Radon Transform via Fourier Slice Theorem - reconstructed images

PHANTOM

90 ANGLES



720 ANGLES



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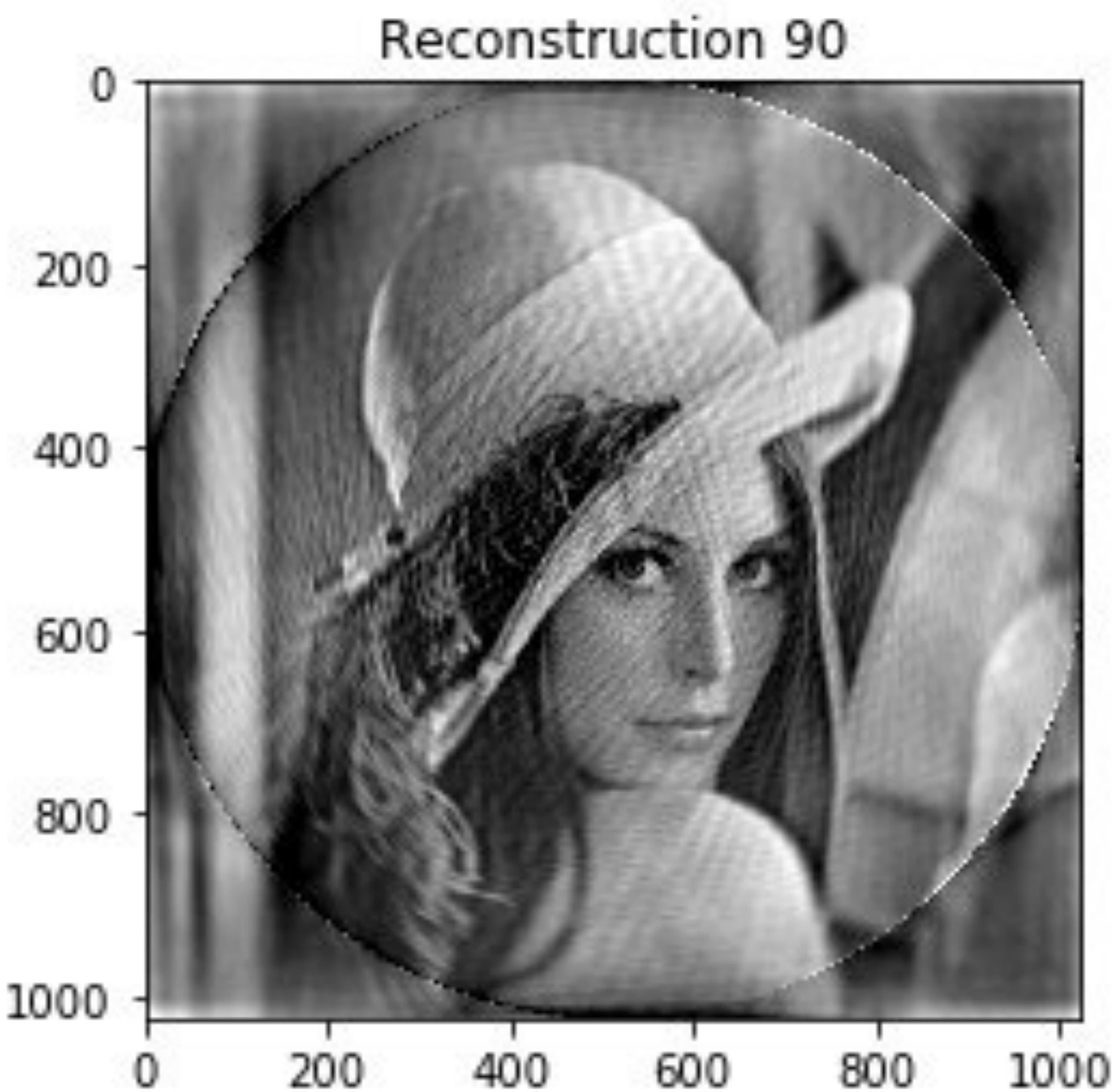
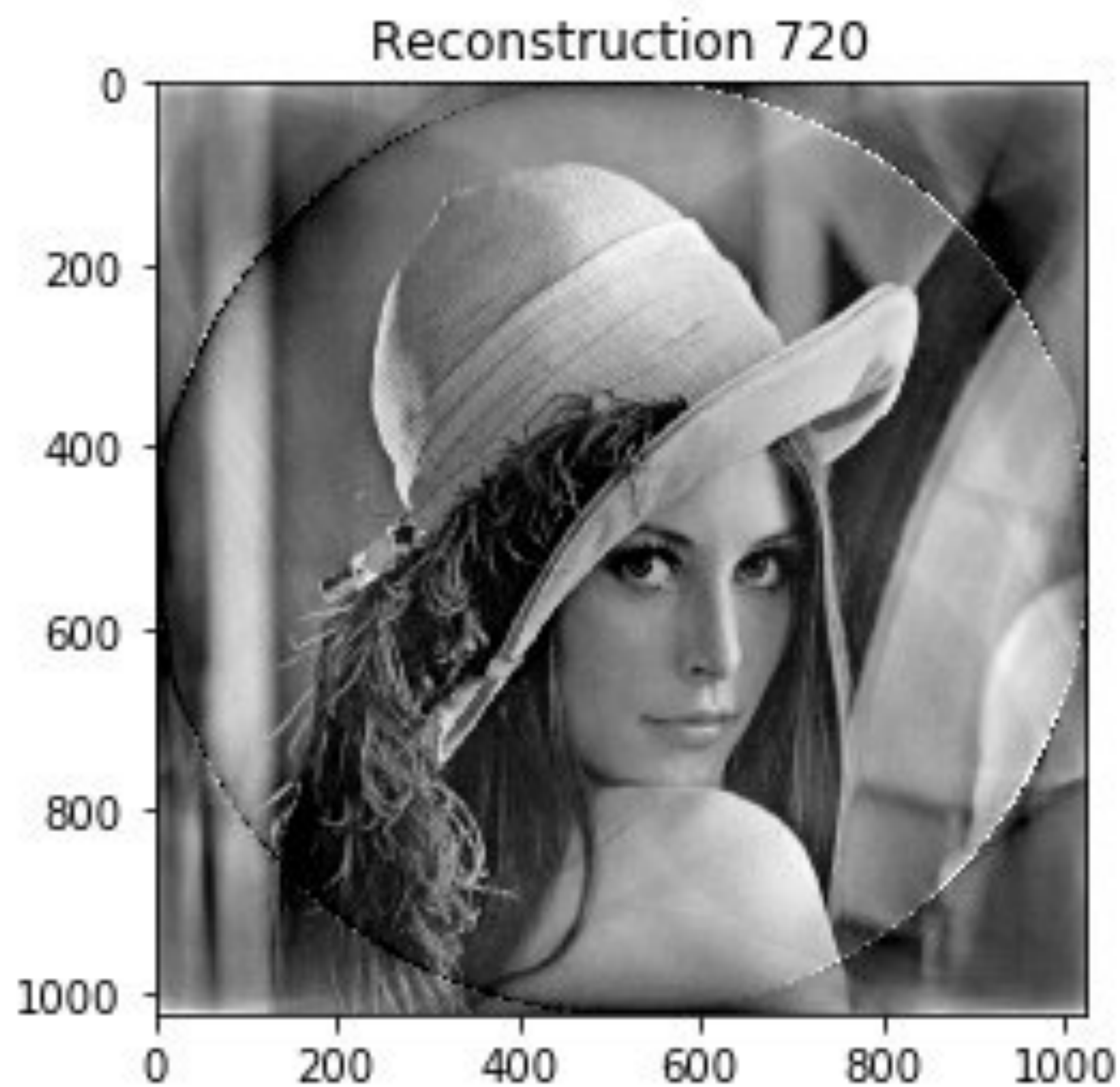
Inverse Radon Transform via Fourier Slice Theorem - reconstructed images

LENA

90 ANGLES



720 ANGLES



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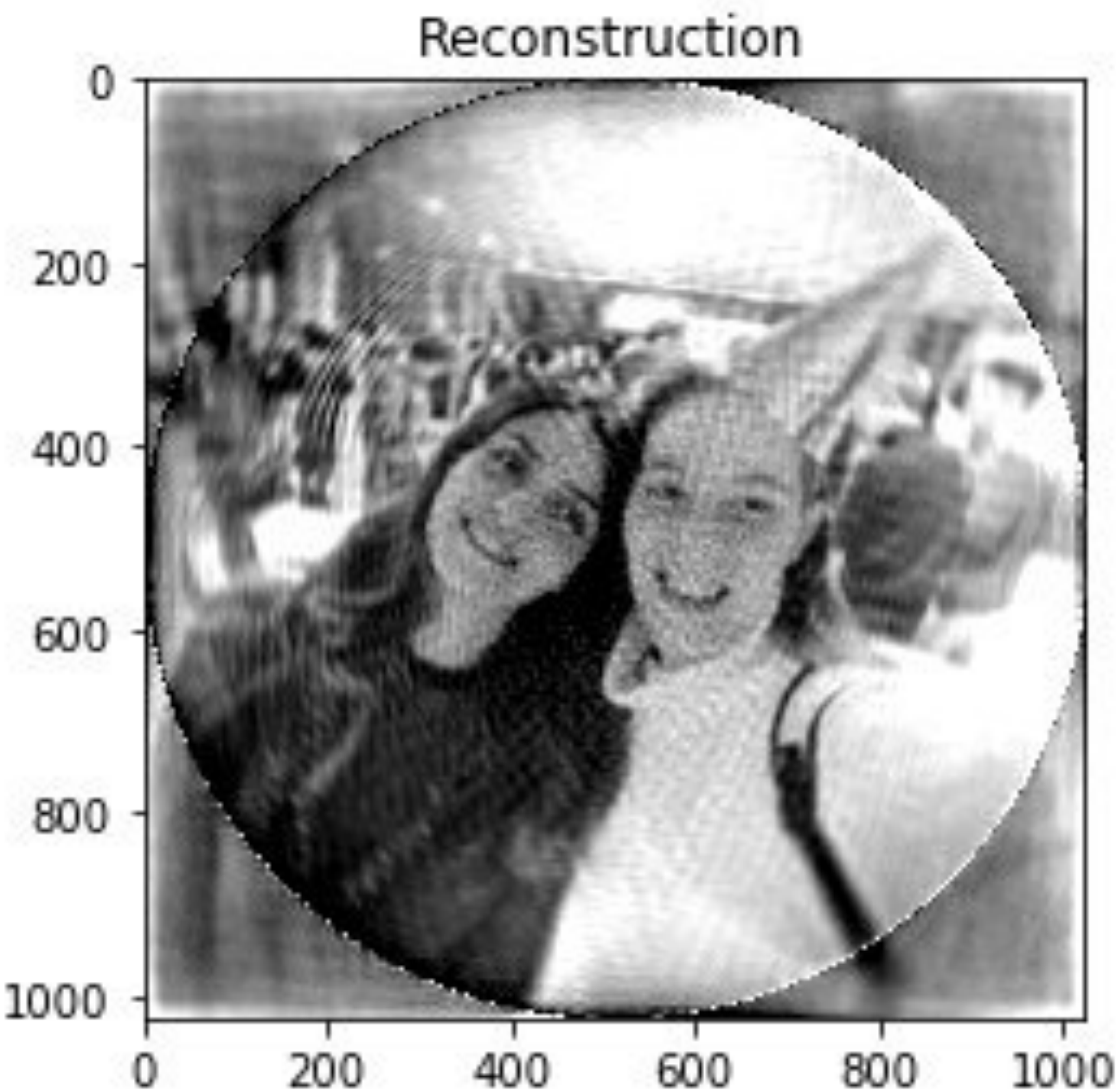
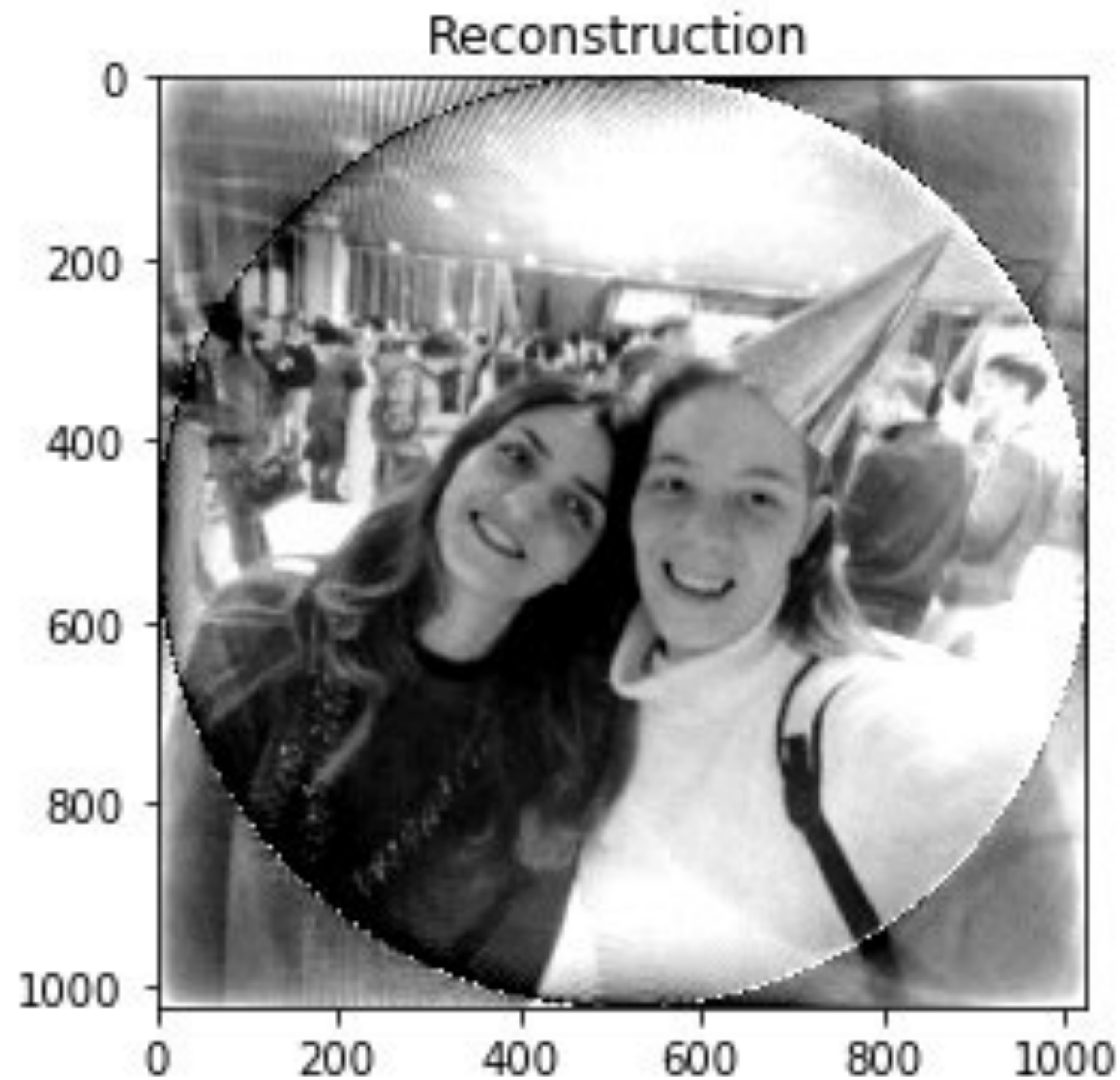
Inverse Radon Transform via Fourier Slice Theorem - reconstructed images

HappyNY

90 ANGLES



720 ANGLES



Approach 2: Inverse Radon Transform as Dual Radon Transform

$g(s, \alpha) - sinogram$

$f(x, y) - image$

$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} z \sin \alpha + s \cos \alpha \\ -z \cos \alpha + s \sin \alpha \end{bmatrix}$

$\hat{R}^*$ - dual Radon transform operator

$L_{\mathbf{x},\alpha}$ denotes line along vector $\mathbf{x}$ and making angle α with axes y

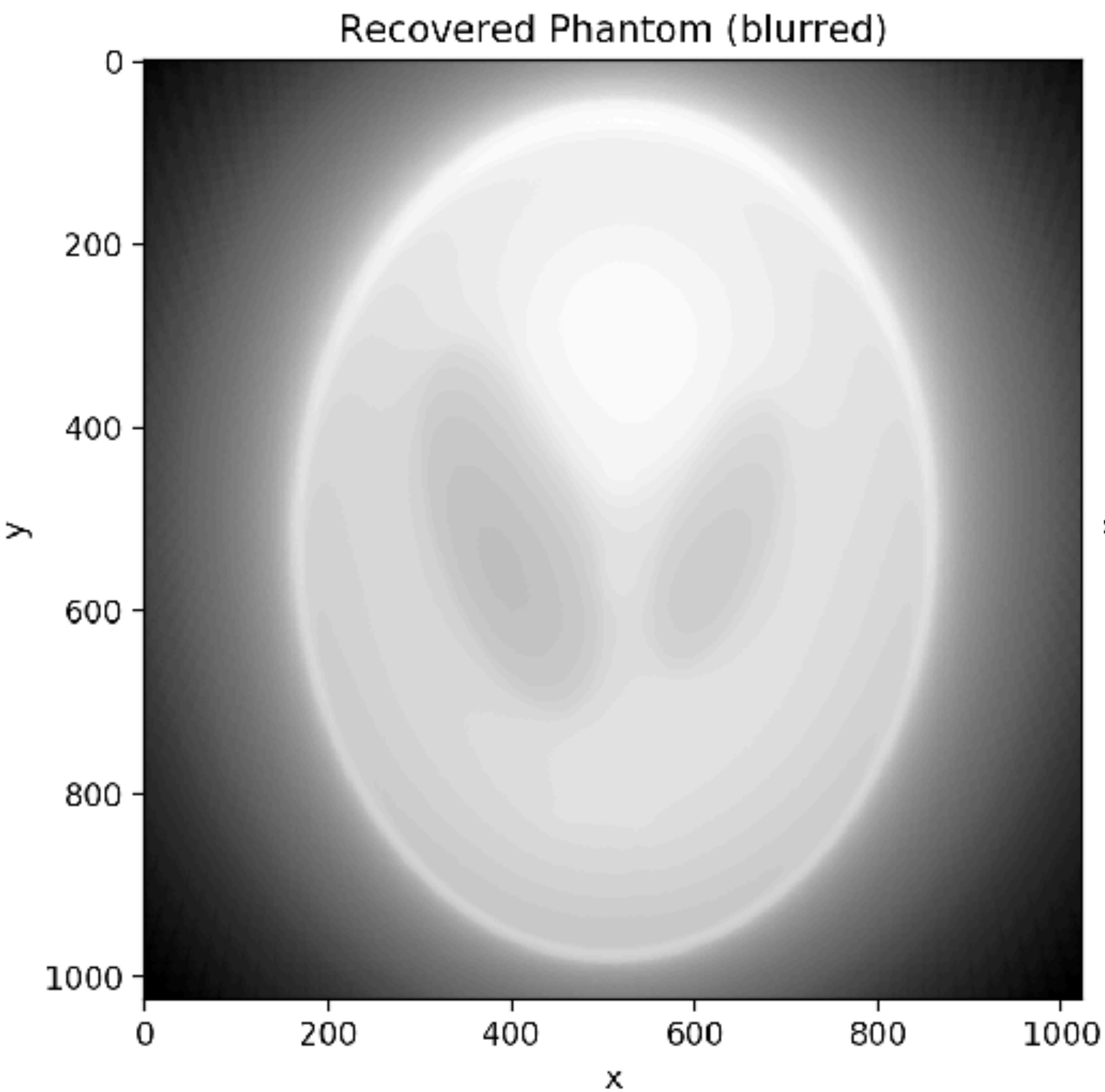
Dual Radon Transform by definition: $$\hat{R}^* g(\mathbf{x}) = \frac{1}{\pi} \int_0^\pi g(L_{\mathbf{x},\alpha}) d\alpha = f(x, y)$$

Why is it dual? $$\langle f, \hat{R}^* g \rangle = \langle \hat{R} f, g \rangle$$

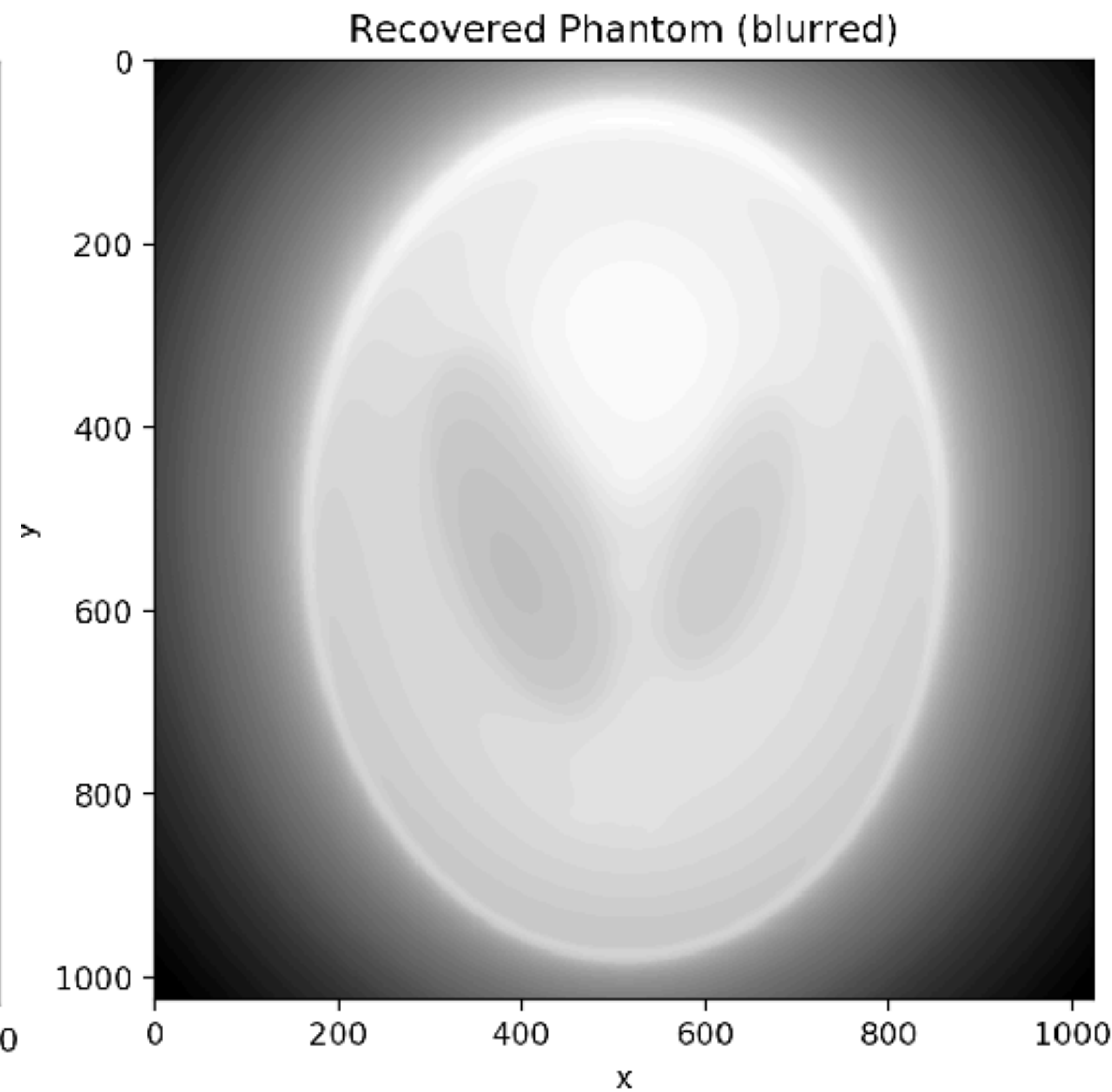
Dual Radon Transform - reconstructed images

PHANTOM

90 ANGLES



720 ANGLES

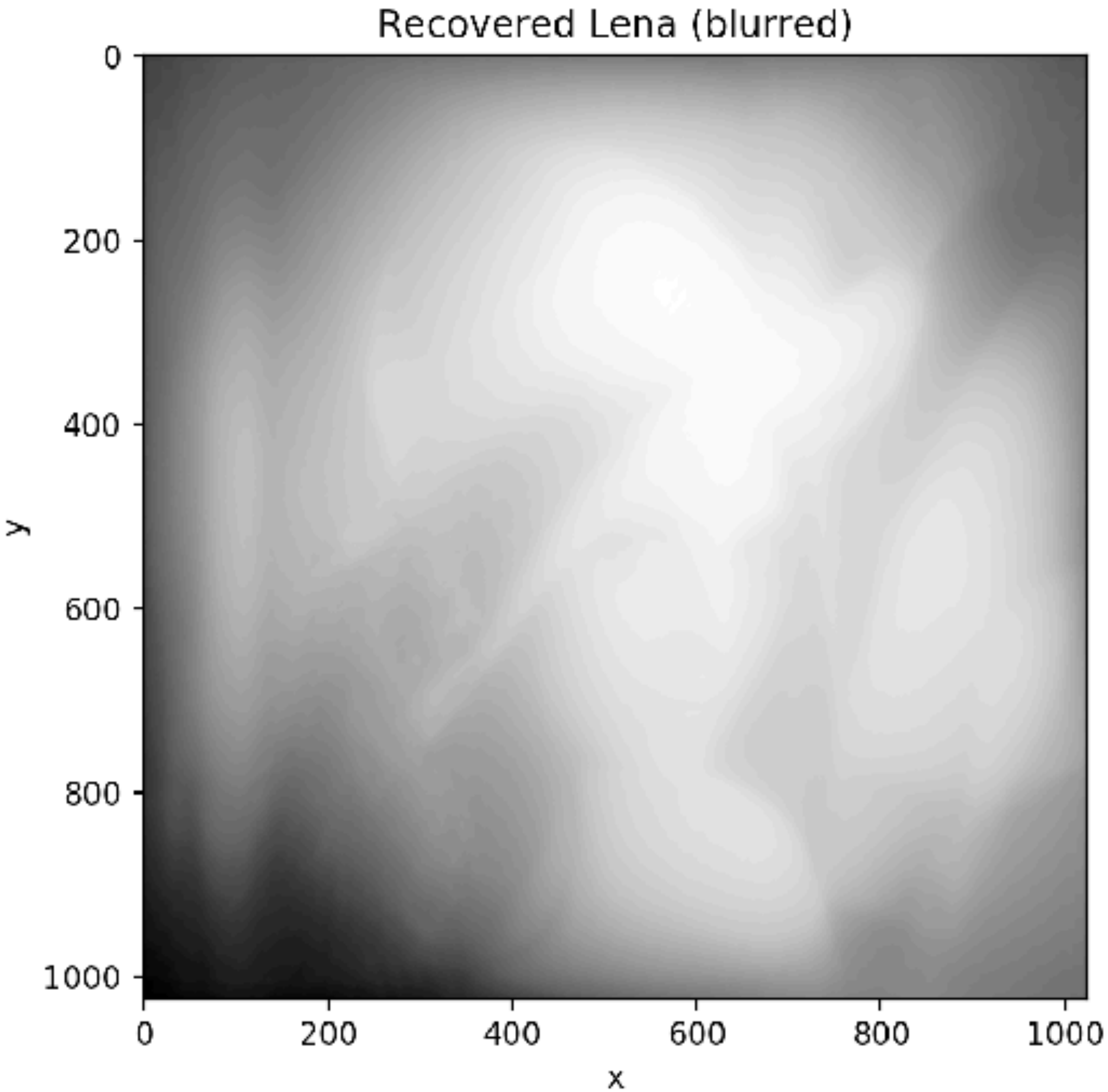


Shapes are more explicit

Dual Radon Transform - reconstructed images

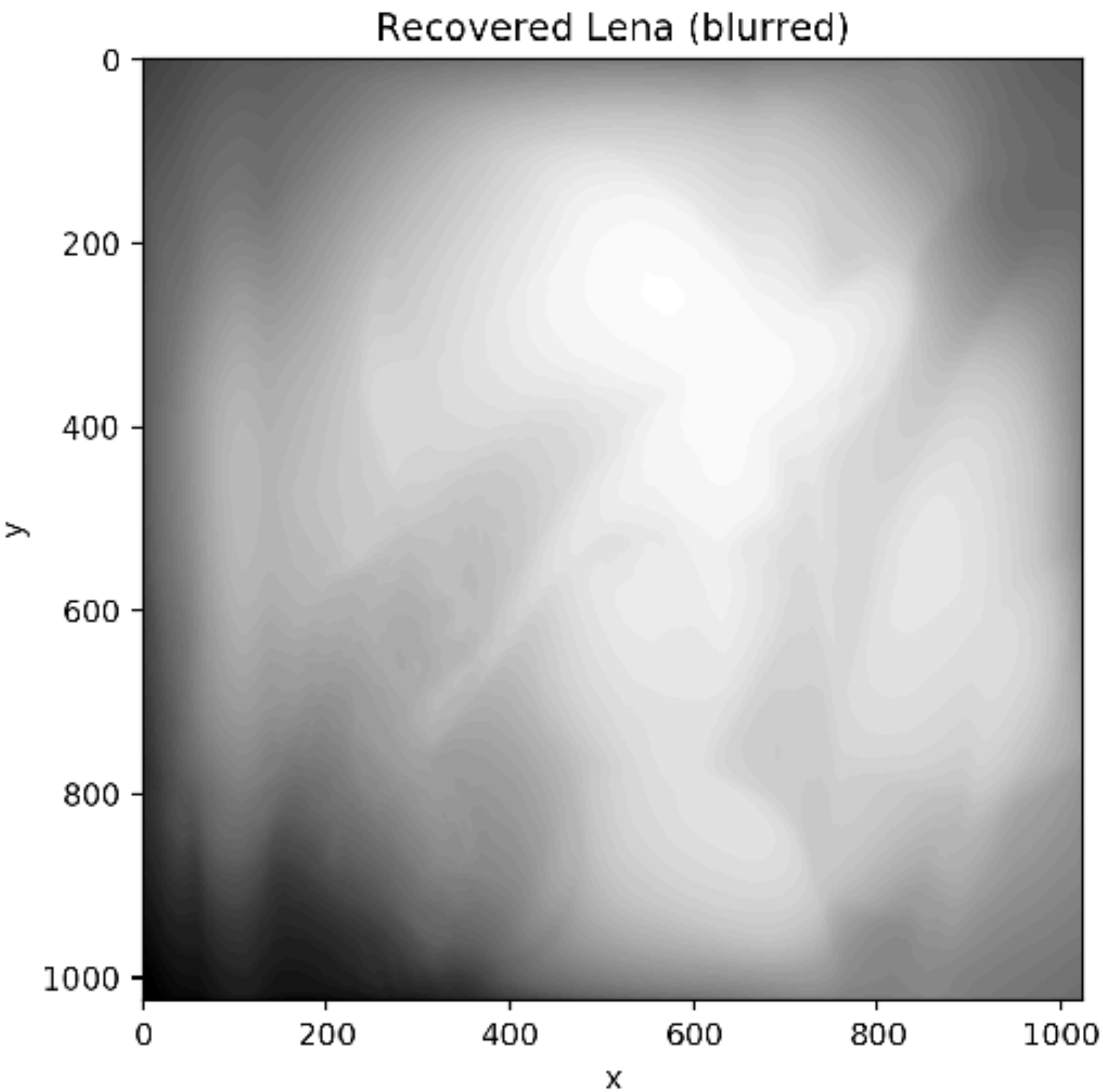
LENA

90 ANGLES



Brighter

720 ANGLES



Darker, more recognizable shapes

Dual Radon Transform - reconstructed images

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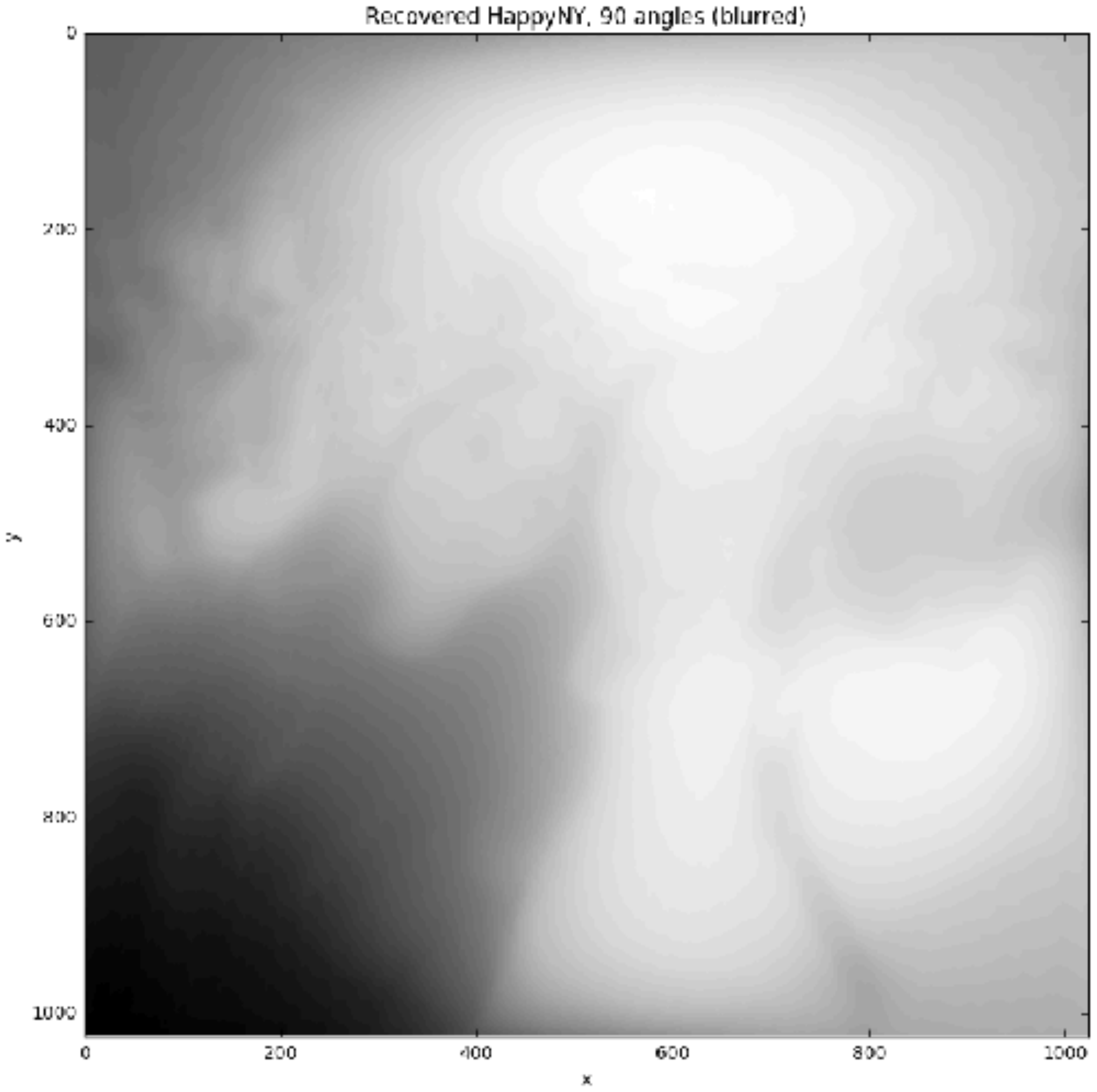
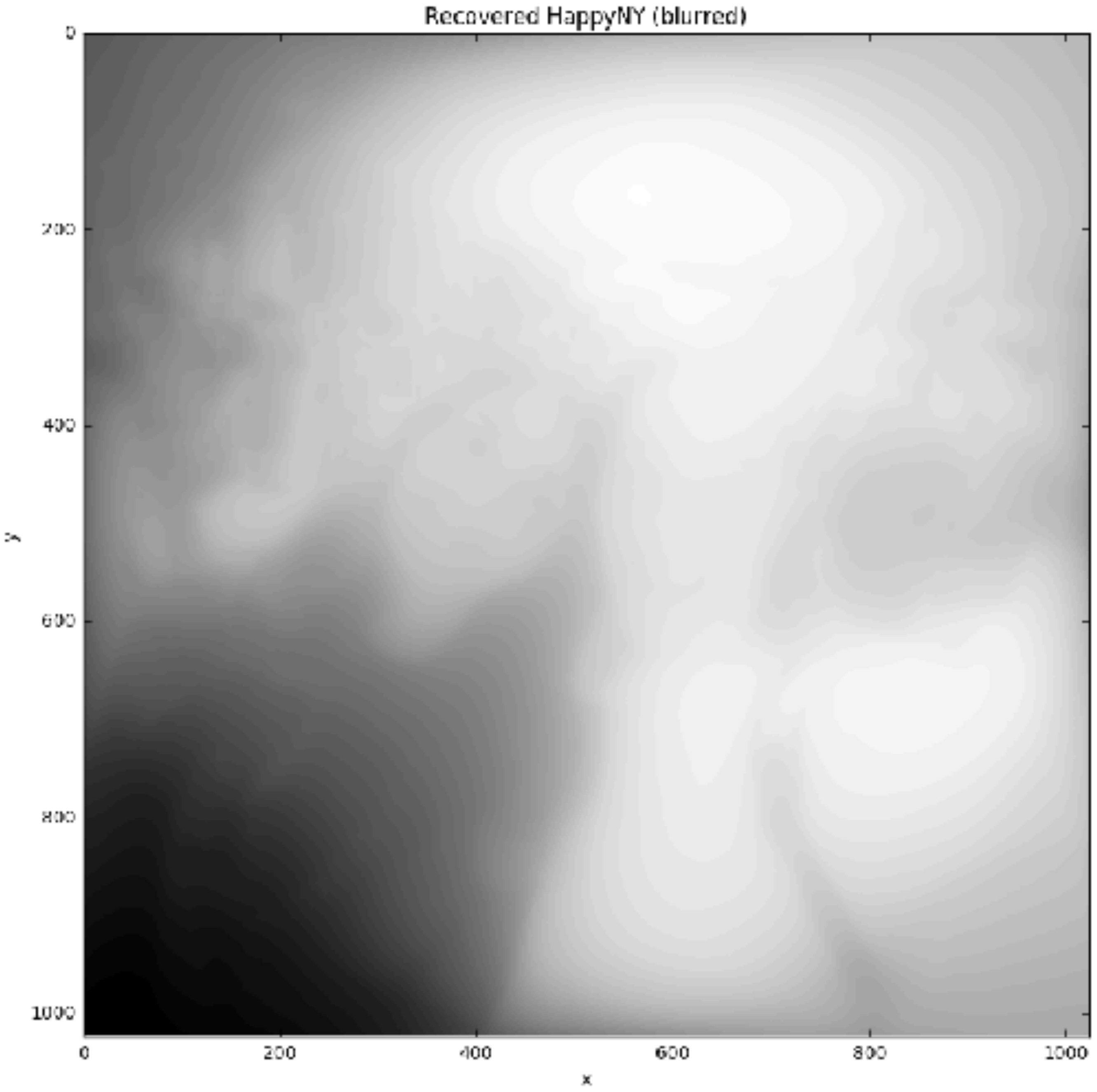
90 ANGLES



720 ANGLES



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Dual Radon Transform - problem

There is a problem:

Reconstructed images are very blurred.

Blur has low frequency nature.

The intuition, based on the fact, leads to the most nature way for deblurring: filtering. We can apply high-pass filter to cut off low frequency components on the reconstructed image.

Actually, there is more common and convenient way, called *Hilbert transform*.

Hilbert Transform - image deblurring

$\hat{H}_s$ - Hilbert transform operator

$\frac{d}{ds}$ - differentiation by distance to slice operator

$f(x, y) = f$ - image

$\hat{R}^*$ - Radon back projection operator

$\hat{R}$ - Direct Radon transform operator

Filtered Back Projection (FBP) :

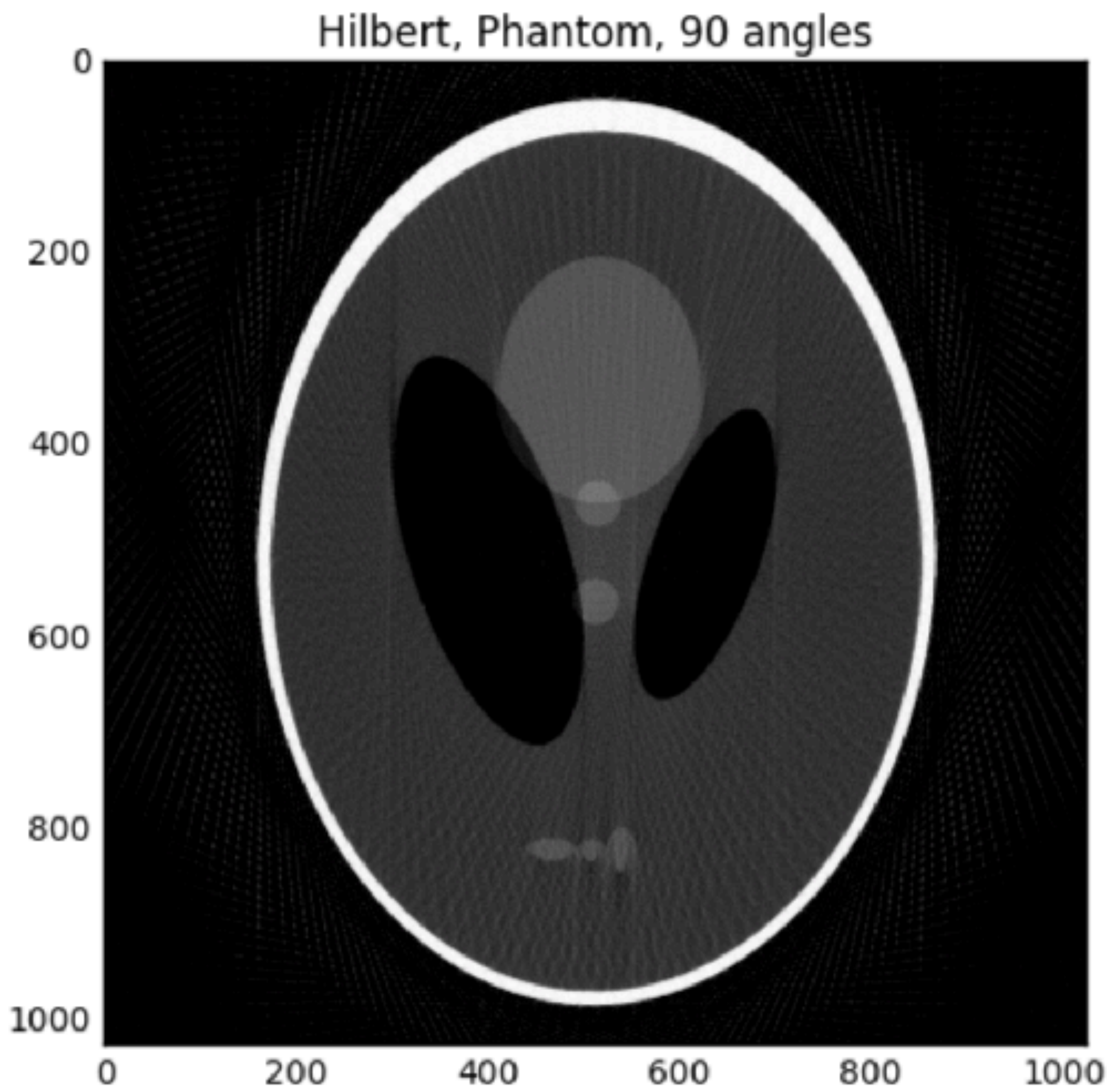
$$\frac{1}{2}\hat{R}^*\hat{H}_s\hat{\frac{d}{ds}}(\hat{R}f) = f$$

This formula is exactly the algorithm for implementation!

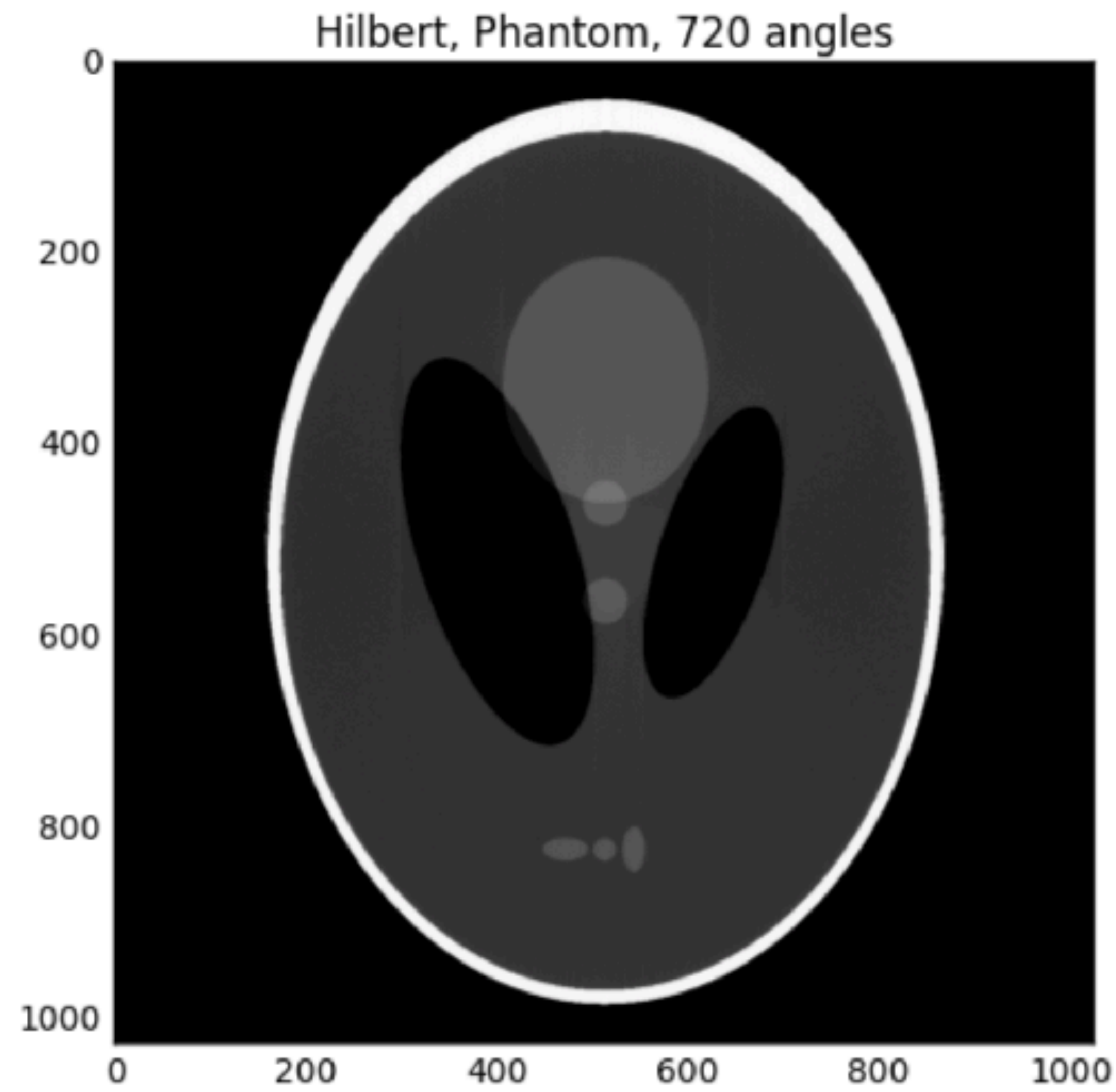
Hilbert Transform - deblurred images

PHANTOM

90 ANGLES



720 ANGLES



There are no 'lost angles', because the image has black background

Piecewise-constant-nature image is well-reconstructable even with lack of observations

Hilbert Transform - deblurred images

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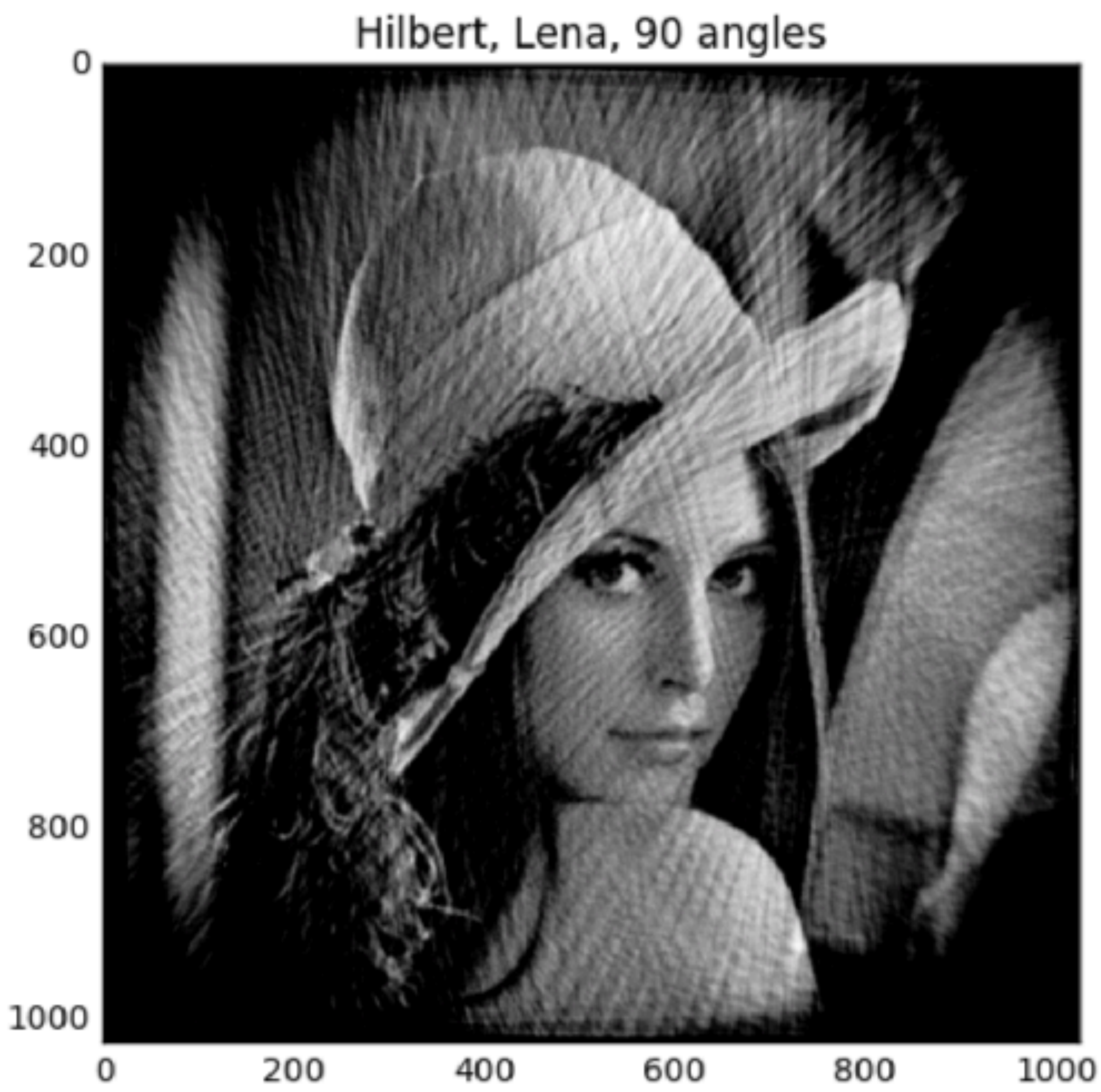
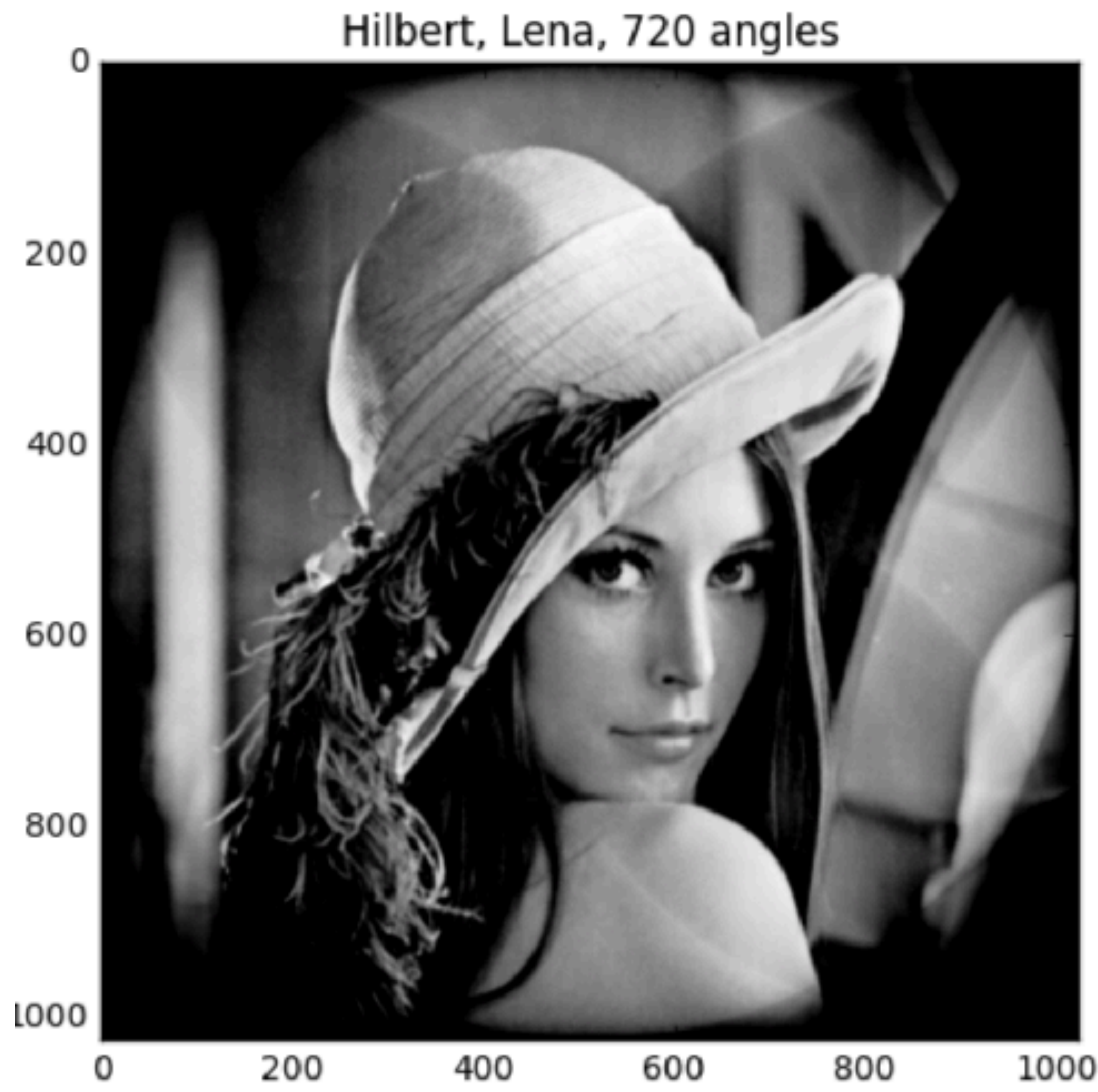
90 ANGLES



720 ANGLES



Hilbert Transform - deblurred images

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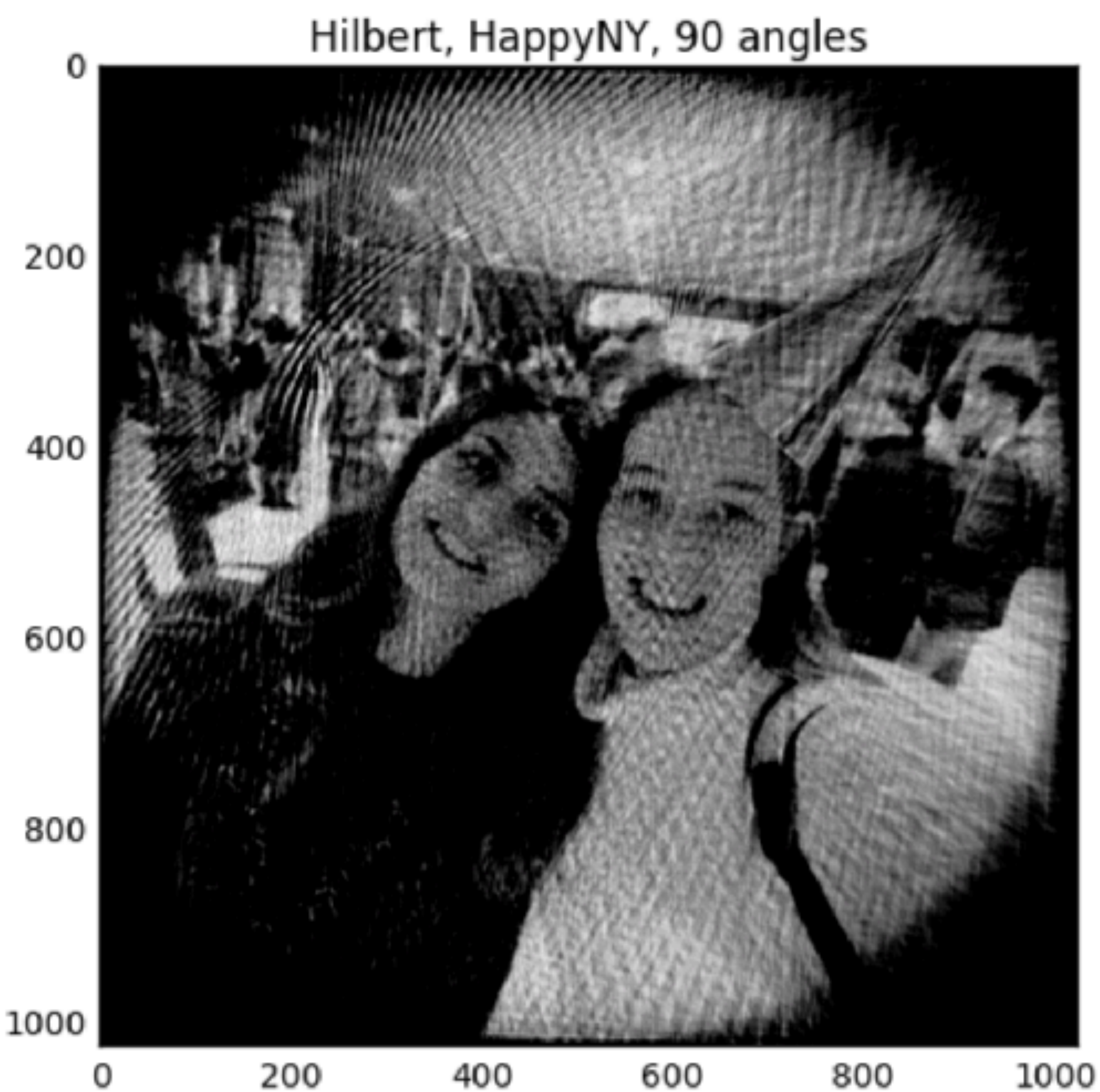
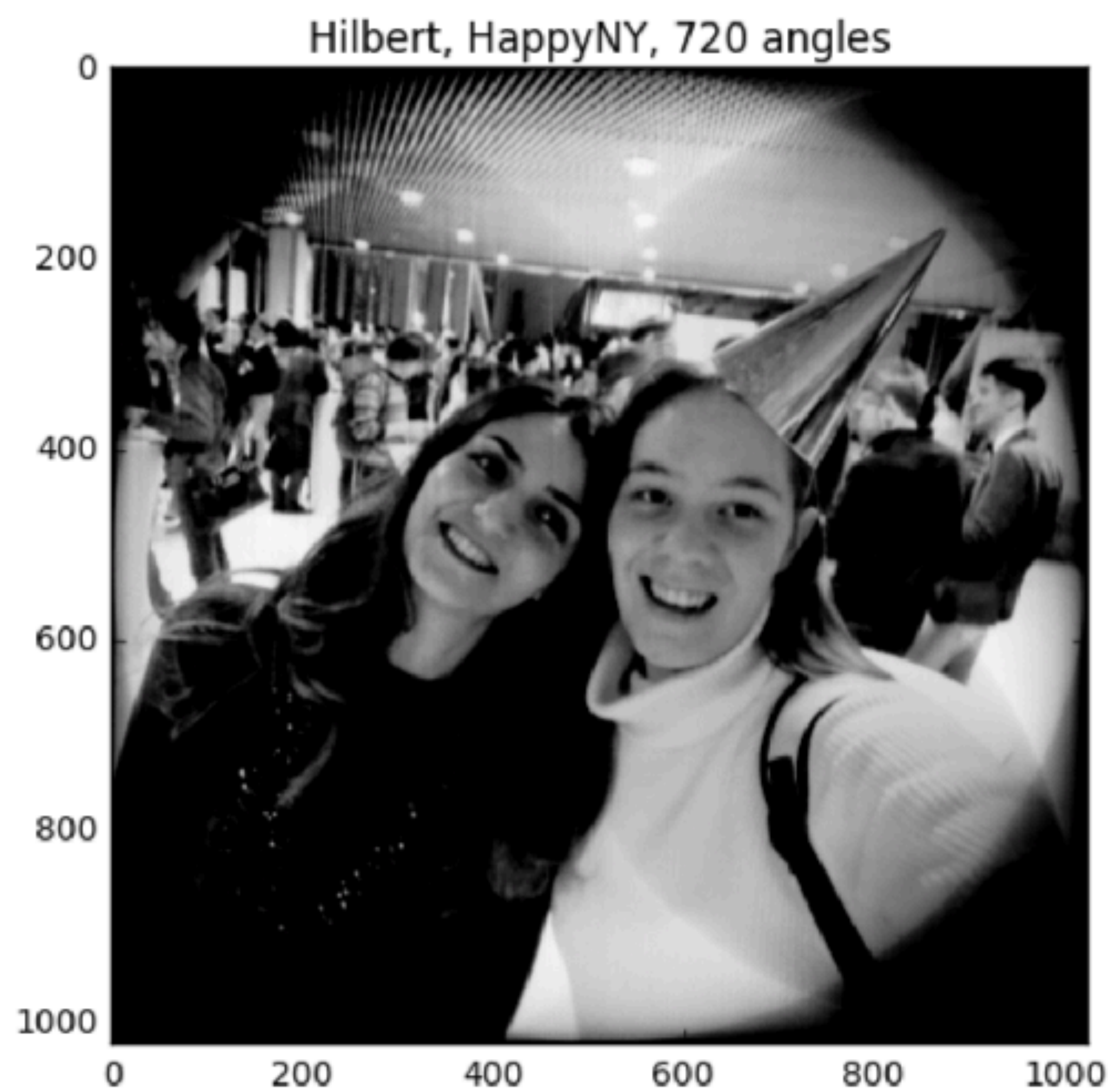
90 ANGLES



720 ANGLES



Algebraic reconstruction technique

Damping of *i*-th X-ray through domain:

$$b_i = \int_{\text{ray}_i} \chi(\mathbf{s})\, d\ell, \quad \chi(\mathbf{s}) = \text{attenuation coef.}$$

Assume $\chi(\mathbf{s})$ is a constant x_j in pixel *j*, leading to:

$$b_i = \sum_j a_{ij}\, x_j, \qquad a_{ij} = \text{length of ray } i \text{ in pixel } j.$$

<u>This leads to a large linear system:</u>

$$A\, x = \overline{b} \qquad \overline{b} = b + e$$

"Geometry"

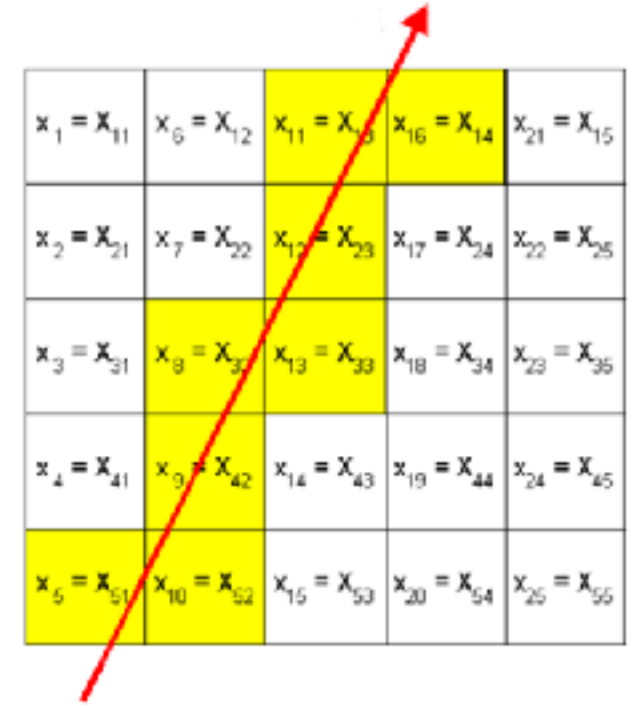
Image

Projections

Noise

$b = A\,\bar{x}$

$\bar{x} = \text{exact image}$



Sparse matrix!!!

- In *scikit-image* package is used so called Algebraic Reconstruction Technique
- This approach is led by application of iterative methods (Kaczmarz' method)
- Basic property: solution will approach a least-squares solution of the equation set
- Good reconstruction normally obtained in a single iteration - computationally effective
- More iterations - **more high frequency noise and less low frequency** (mean squared error)
- Number of iteration - set manually

Algebraic reconstruction technique

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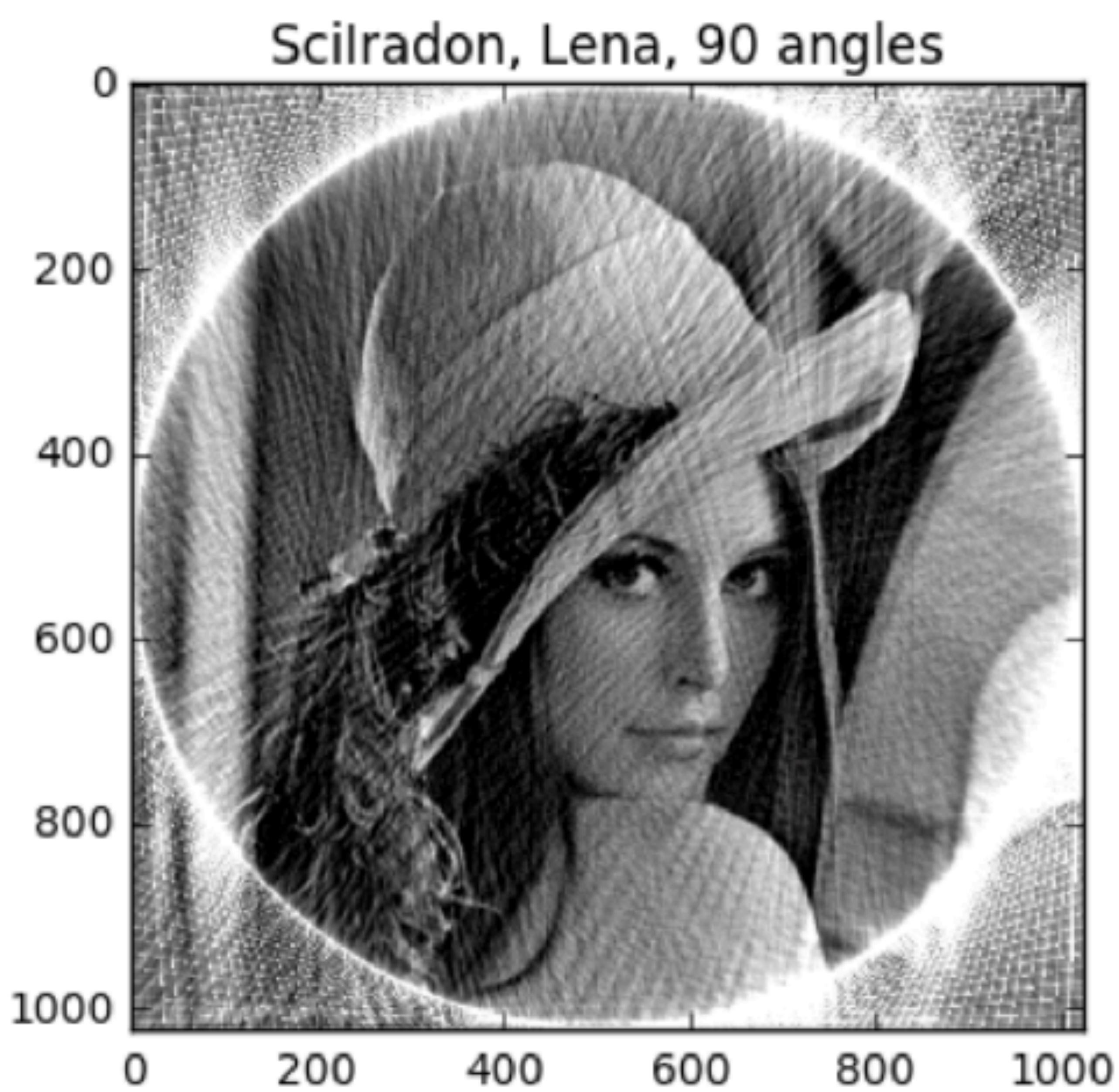
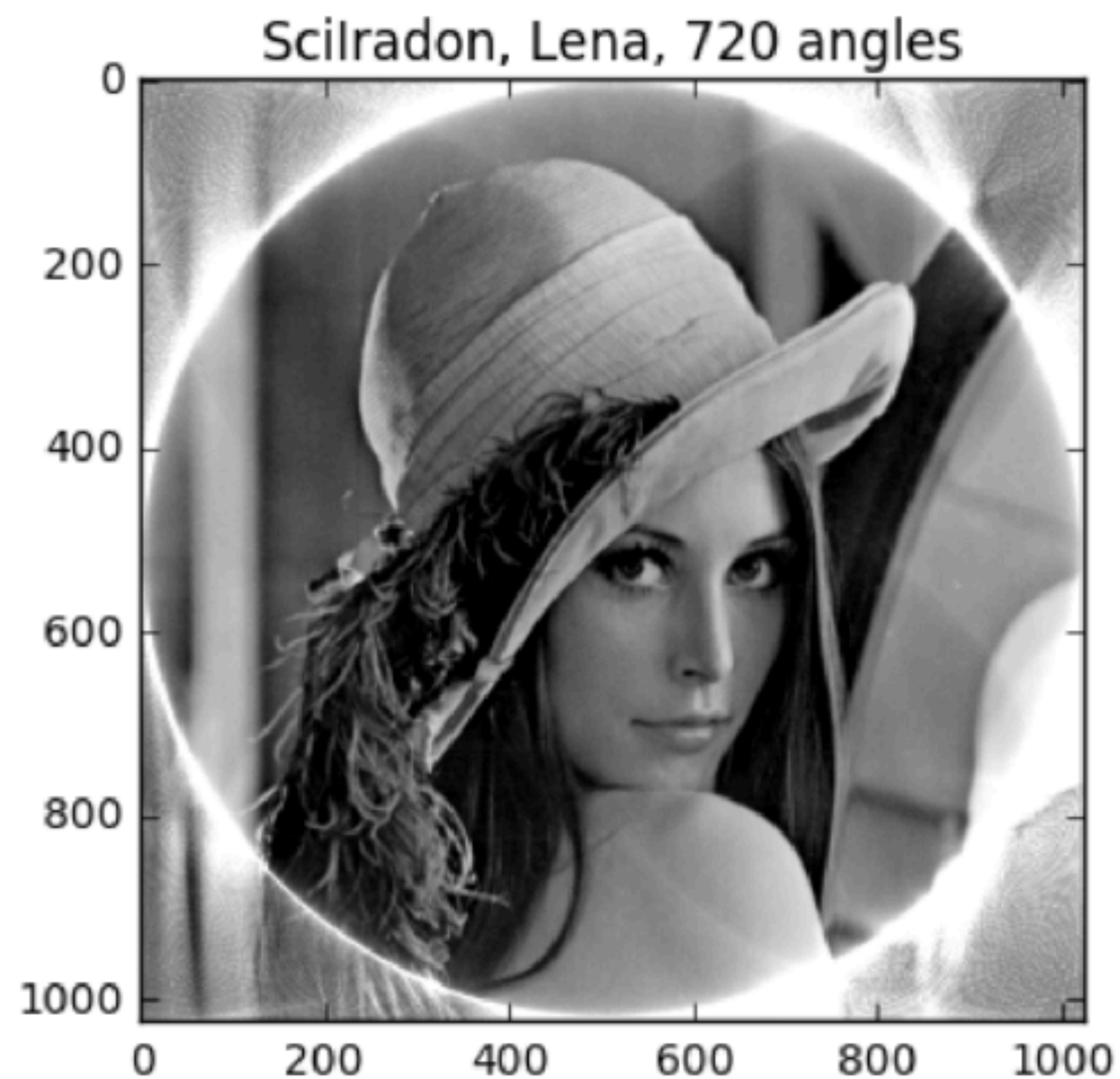
90 ANGLES



720 ANGLES



Accuracy and performance comparison

Our implementation

Inverse via Furier ST

	Full	Restricted
90	0.2830	0.1704
200	0.2706	0.1349
720	0.2749	0.1651

Hilbert tr. filtering

	Full	Restricted
90	0.61219	0.4583
200	0.5996	0.44130
720	0.59452	0.43768

Accuracy and performance comparison

Scikit

90 Angles

	Recovery time from sonogram	Relative error for norm L2	For the central part
Phantom	14.122	0.43161	0.24284
Lena	14.001	0.68268	0.30209
Happy NY	13.979	0.77730	0.33838

200 Angles

	Recovery time from sonogram	Relative error for norm L2	For the central part
Phantom	14.5628	0.2733	0.1753
Lena	14.7636	0.6127	0.2535
Happy NY	14.4507	0.6811	0.2950

720 Angles

	Relative error for norm L2	For the central part
Phantom	0.21424	0.1333
Lena	0.58401	0.2438
Happy NY	0.6398	0.2826

Summary

- Slice Fourier Theorem - good reconstruction, built-in filter (but quite slow)
- Dual Radon Transform - blurred image - Hilbert transform for deblurring - quite accurate reconstruction
- Algebraic approach - the fastest, very accurate, but should be manually adaptated