Inverse design of 2D wave devices with optimization

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Problem

- Modeling of interaction of electromagnetic waves in conducting medium
- Representation of electromagnetic waves animations
- Topological optimization of electromagnetic wave based devices

Existing methods

- Direct solution of Maxwell equations
- Experimental observing
- Numerical methods

Who cares?

- Physics based studying of electromagnetic waves entity
- Radio photonics
- Topological optimization of size and performance of photonics devices
  - HUAWEI, CISCO, INTEL, SAMSUNG, IBM

Main wave integral equations (Integral form of Helmholtz equation)

Required field

Green function

Permittivity

Electromagnetic intensity distribution in vacuum

$$E(x) - k^2 \int G(x-\tilde{x})\left(\varepsilon(\tilde{x}) - 1\right)E(\tilde{x})\,d\tilde{x} = F(x)$$

Wavenumber

$$G(r) = \frac{i}{4} H_0^{(1)}(k \cdot r)$$

Hankel function

Wave equation descretization (One type of materials)

$$Ax = f$$

Constatnt

$$A = I - k^2\left(\varepsilon - 1\right)G \cdot diag(m)$$

Mask: {0,1}

$$x = vect(E)$$

$$f = vect(F)$$

Non convex topological optimization problem

$$\max_{m} \left\| x(m) \right\|_{\infty}$$

$$\text{s.t} \quad \left(I - k^2 (\varepsilon - 1) G \cdot diag(m) \right) x = f$$

$$m \in \{0,1\}$$

Penalization of optimization problem

$$\max_{m} \left\| x(m) \right\|_{\infty} - c \cdot m^{T} \left(1 - m\right)$$

$$\text{s.t} \quad \left(I - k^{2}\left(\varepsilon - 1\right) G \cdot diag(m)\right) x = f$$

$$0 \le m \le 1$$

Optimization acceleration (Jacobi matrix)

$$A \cdot J_m(x) = B$$

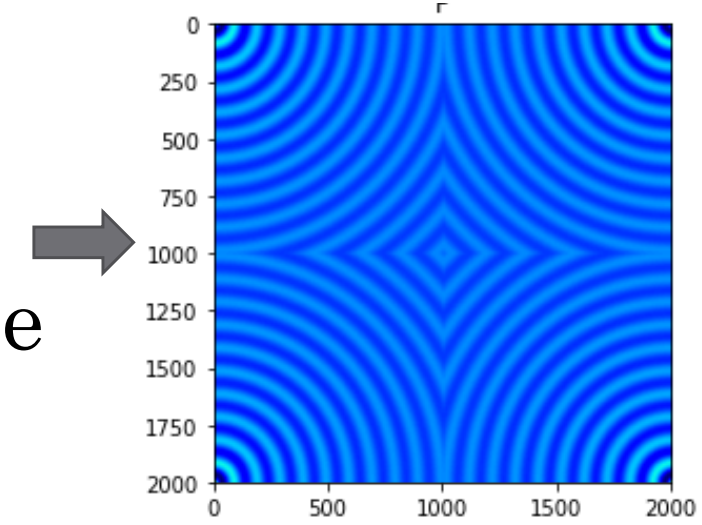
$$A = I - k^2(\varepsilon - 1)G \cdot diag(m)$$

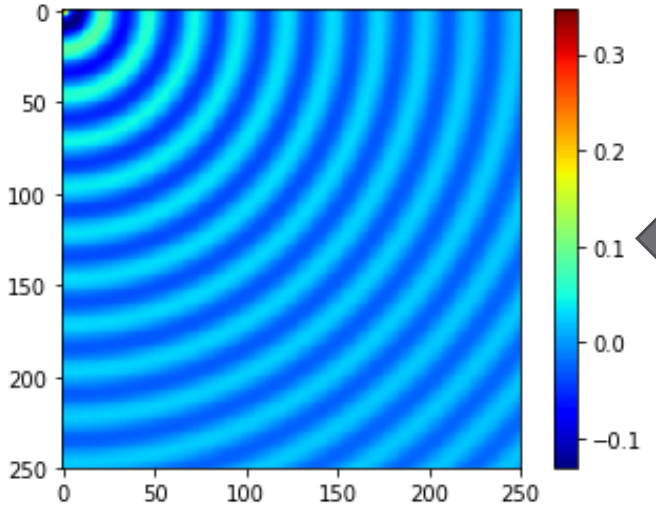
$$B = k^2(\varepsilon - 1) \cdot diag(m)$$

FFT matvec for G product

- Size G is n^2 x n^2!
- But since G is toeplitz matrix we can use fast FFT matvec
- We don't store full matrix usung FFT matvec linear operator (size ~n x n)

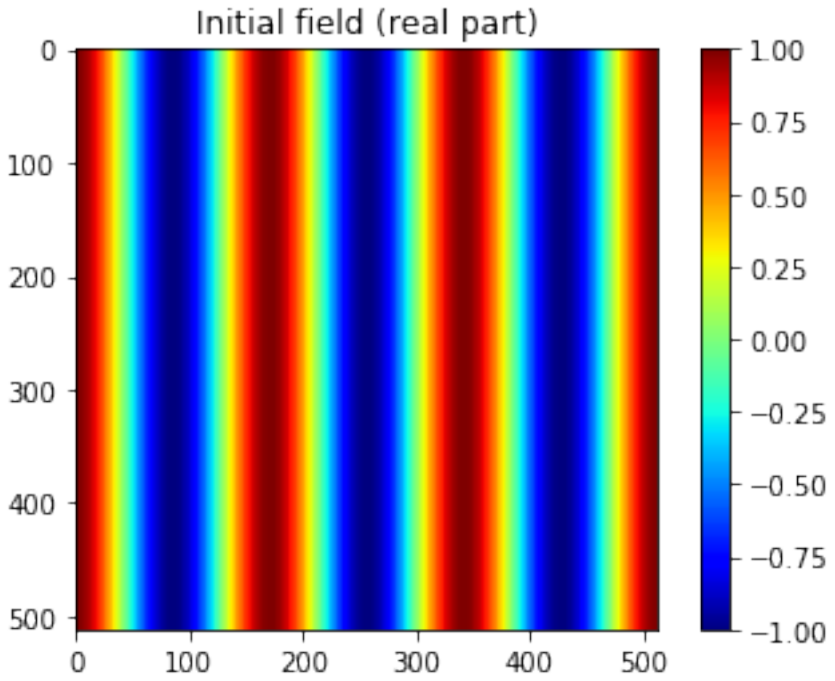
Circulant matrix (Surrogate of G)

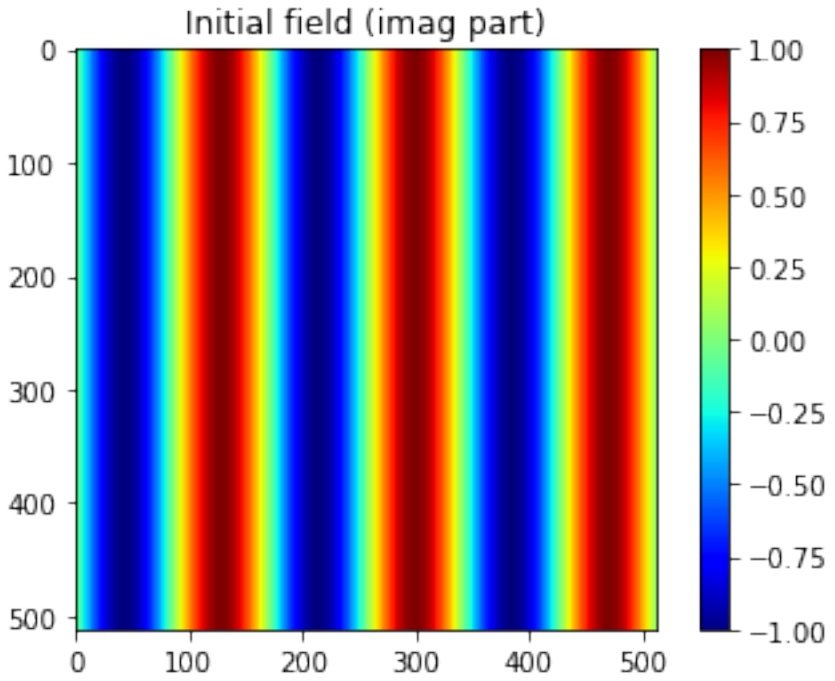




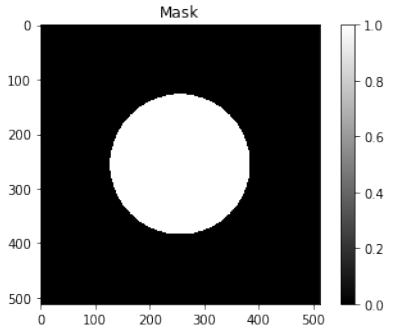
Toeplitz block of G

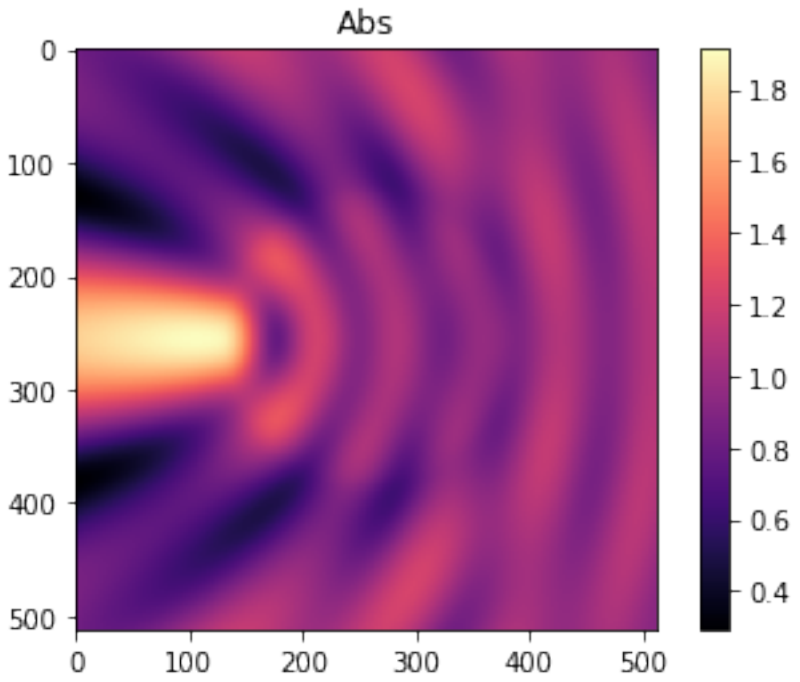
Some examples of work of our solver (in Python). Initialization.

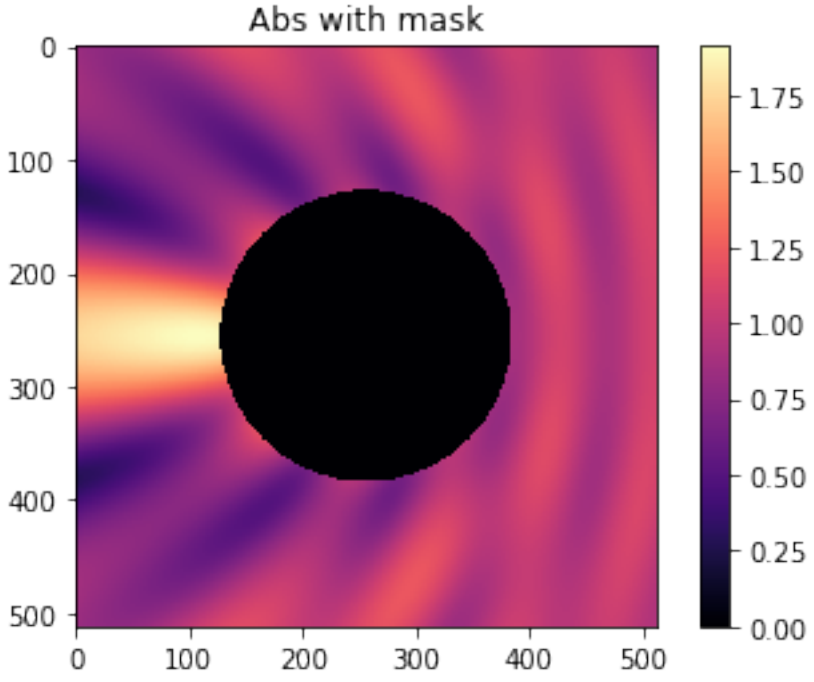
$F =$ 

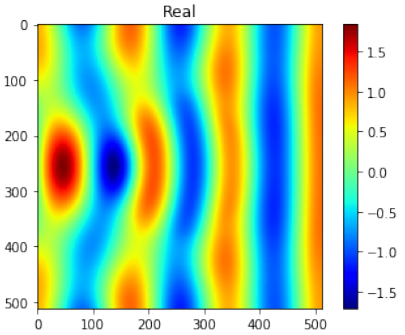
$+ j \cdot$ 

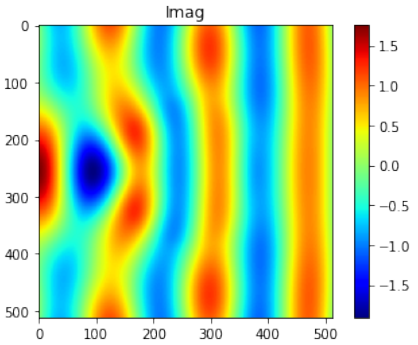
Circular mask ($\varepsilon = 1.5$)



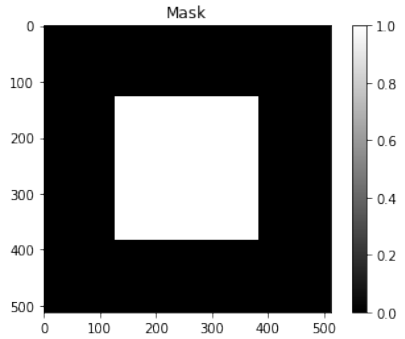


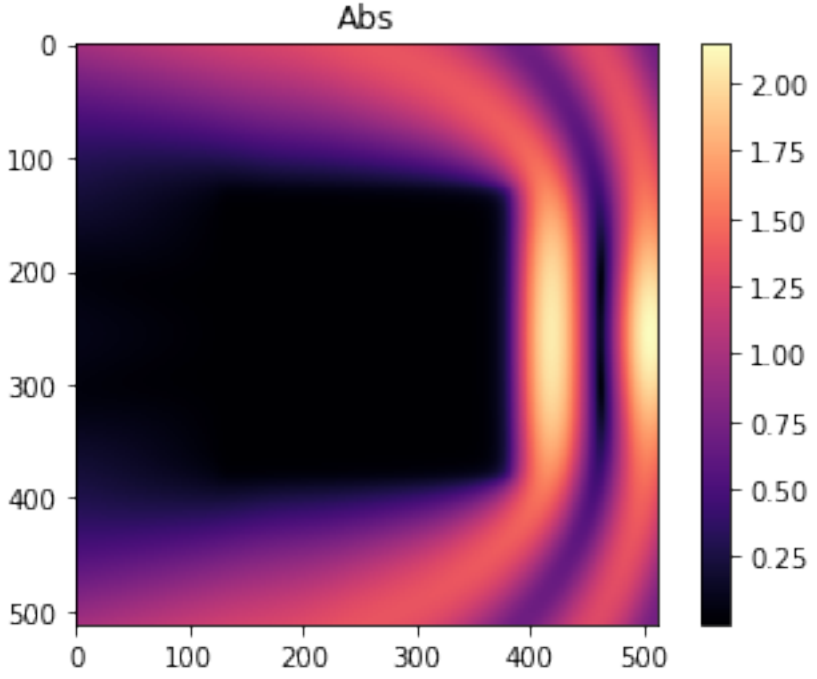


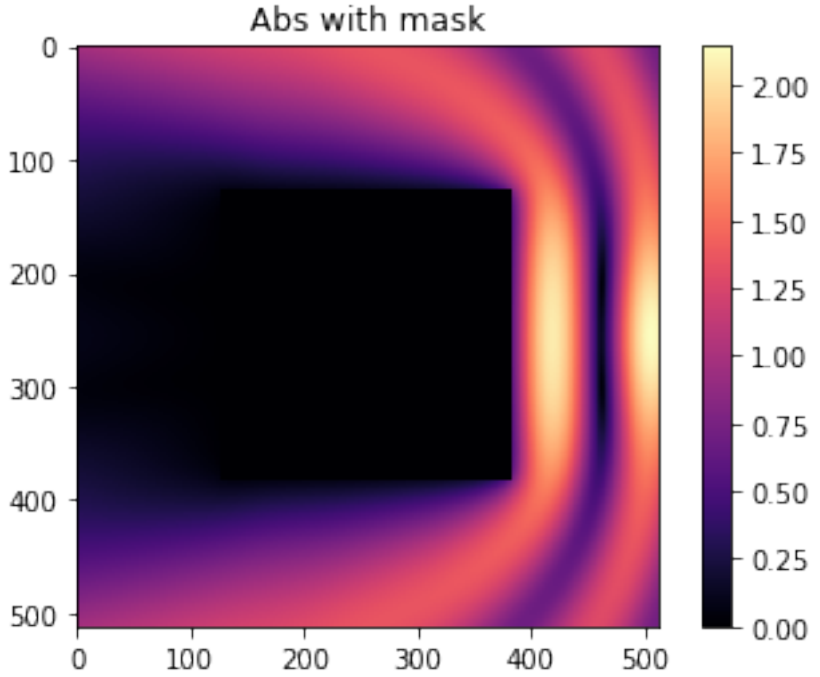


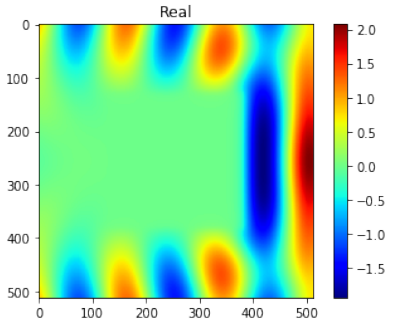


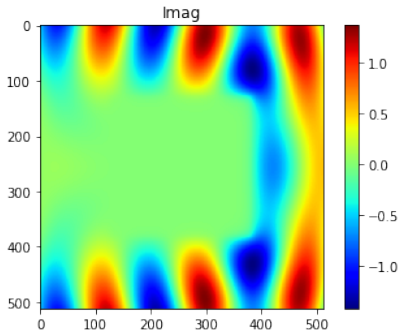
Rectangular mask ($\varepsilon = -10$)



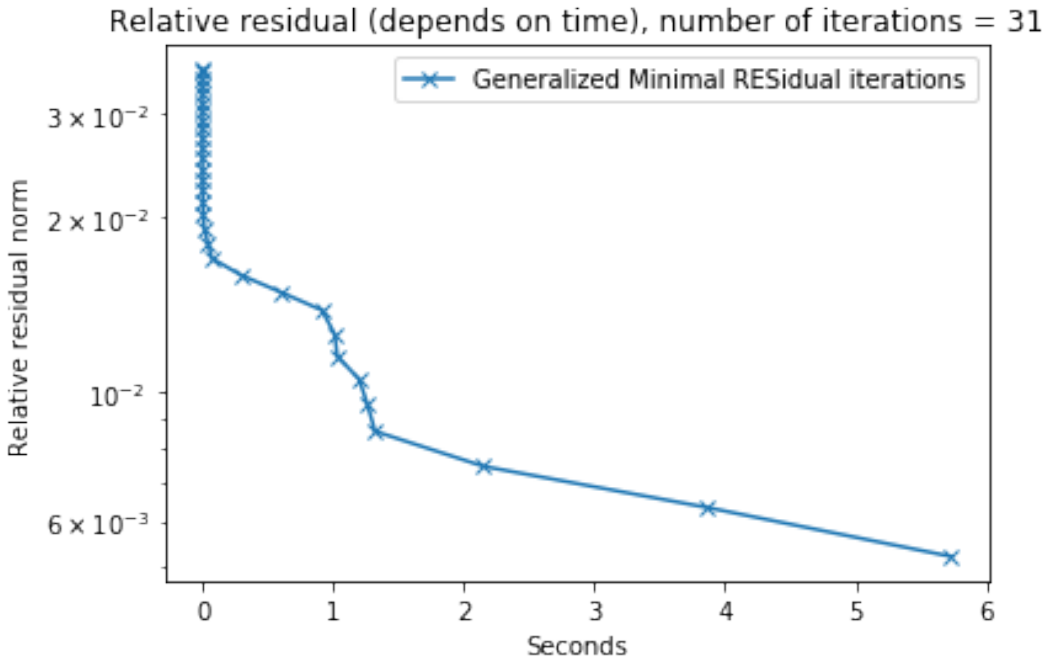


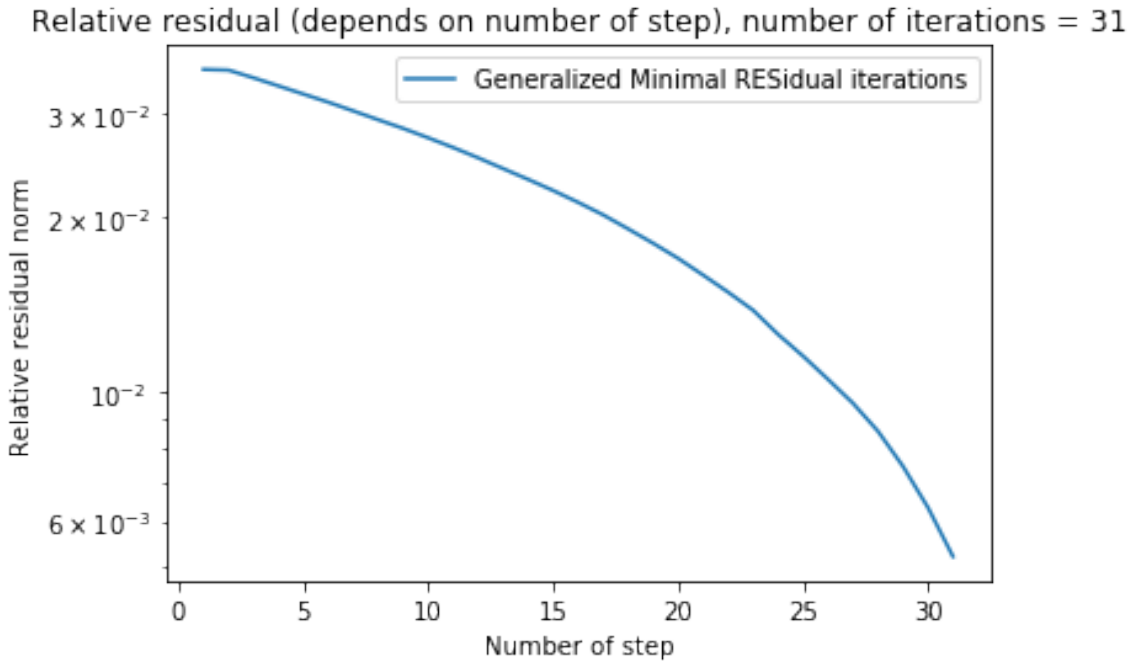






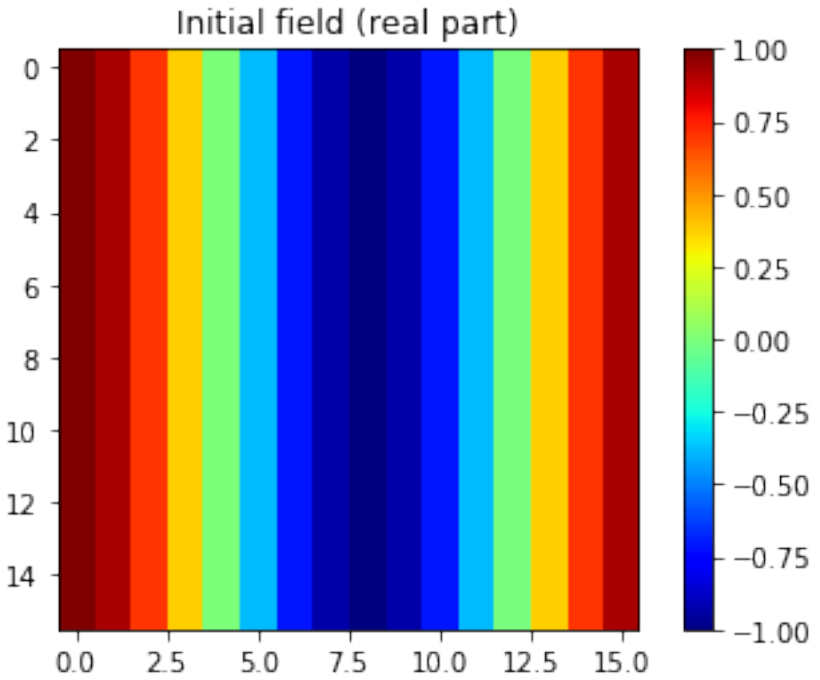
Converges of GMRES Solver (grid 512×512)

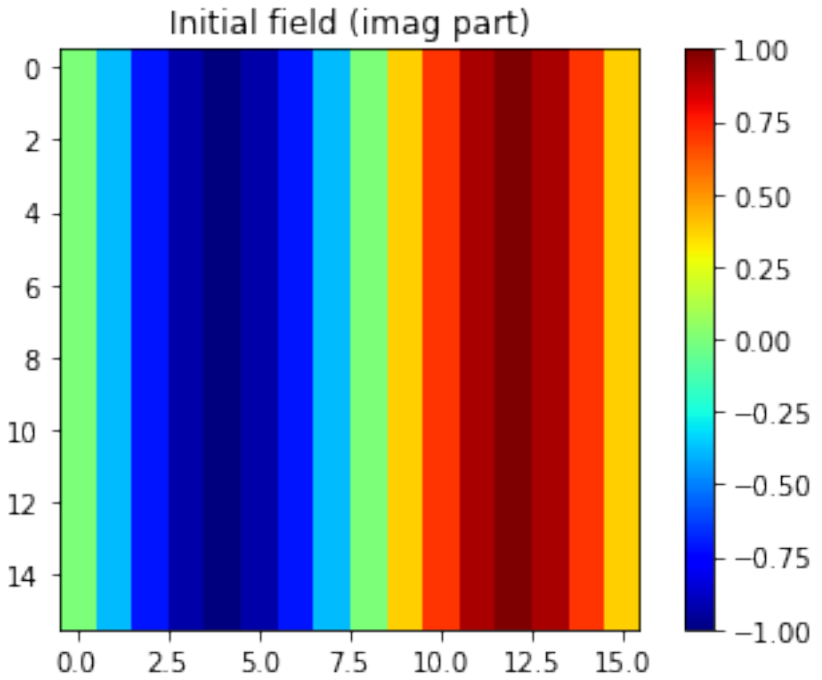




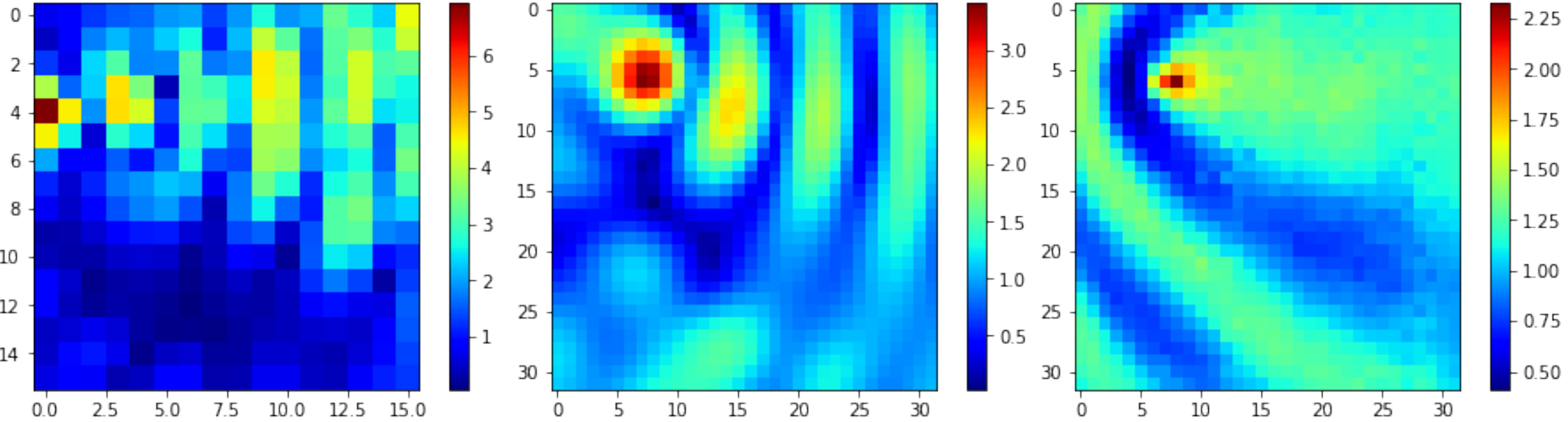
Results of optimization. Initialization

- For this task we have used grid 16×16, since optimization is more hard computing task

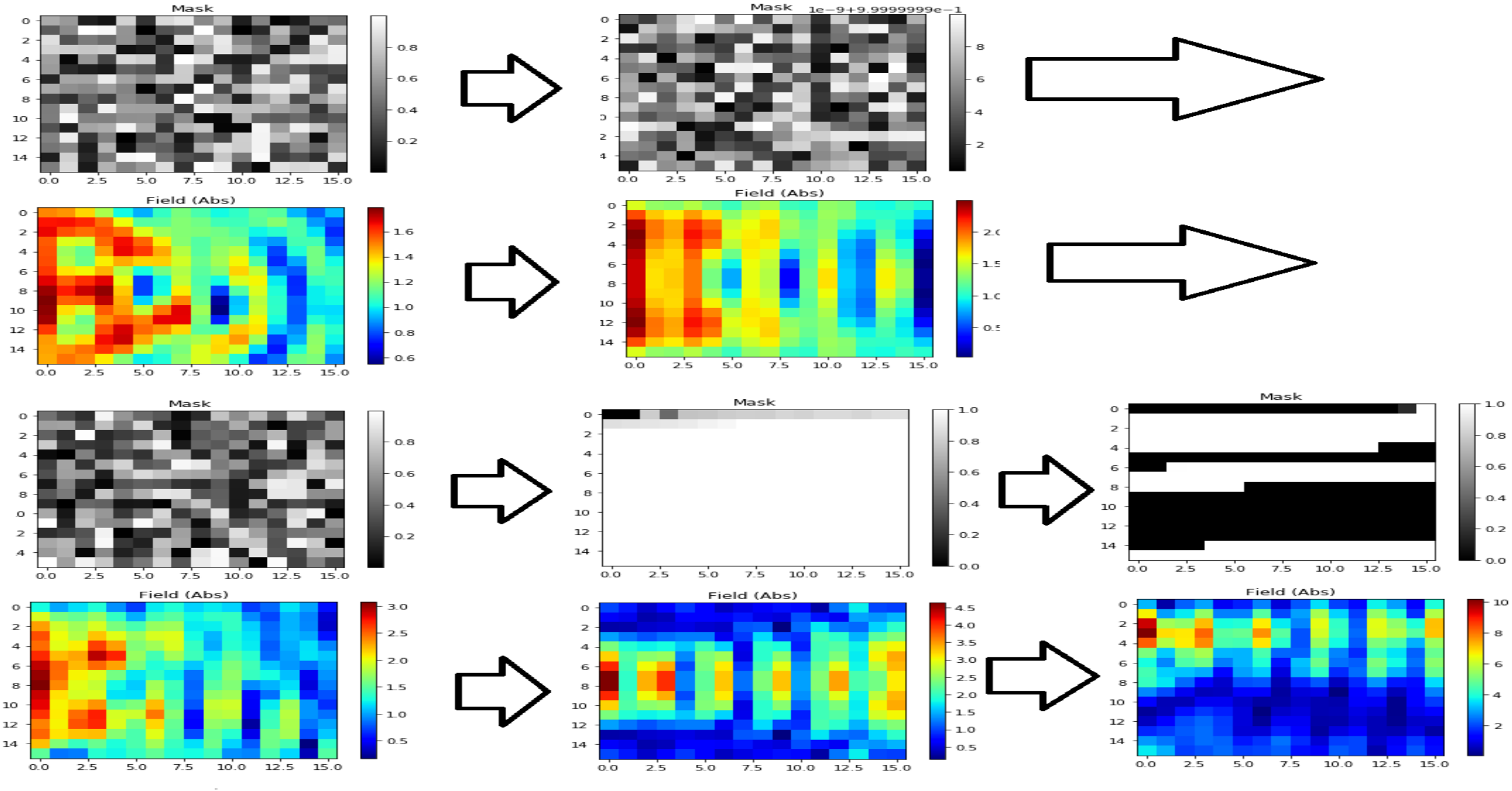
$F =$ 

$+ j \cdot$ 

Some optimization examples on continuous **ε** grid



Discrete mask case



Thank you