Air Cargo Capacity Management

Team members: Alexander Nekrashevich, Aibek Alanov, Filippos Kokkinos, Sergey Pozdnyakov, Dmitiy Salnikov
With Cherry Flavor

Contents

Problem Formulation

Our aim is to forecast the prices p_t which maximize our revenue $f_t(b)$ (depends on the amound of booking b) from cargo transporation at each time $t = 0, \dots, T$. We can write down the dynamic relation for $f_t(b)$:

$$f_t(b) = \max_{p_t} \mathbb{E}_{\mathbb{D}}\{p_t \mathbb{D}(p_t) + f(t+1)(b + \mathbb{D})\}, \ t = 0, \dots, T-1,$$

$$f_T(b) = \mathbb{E}_{N_1 \dots N_r} \max_{Q_1 \dots Q_r, Q_i \le N_i} \sum_{i \in R} \{-c_i Q_i - c_D(N_i - Q_i) - m_B(b_i - N_i)\}$$

$$\text{subject to} \quad \sum_{\text{all paths i containing edge } e} Q_i \le W_e$$

Solution

1. **Baseline Method.** Conservative approach with no-decline guarantee. Selecting optimal prices.
2. **Deterministic Approach.** Replace random variables with their expected values, and then run maximization problem.
3. **Lagrangian Relaxation.** Described in the next slide.

Lagrangian Relaxation

We can optimize our resulting revenue $f_0(b)$ for all prices $p_t,\ t = 0, \ldots, T$ simultaneously using Lagrange function:

$$\mathcal{L}(\lambda, p) = \sum_{e \in \text{edges}} \lambda_e W_e + \sum_{t=0}^{T-1} \sum_{i=0}^{R} (\mu_i + p_{t,i}) \mathbb{ED}(p_{t,i}) \ \Rightarrow$$

$$\Rightarrow \ G(\lambda) = \sum_{e \in \text{edges}} \lambda_e W_e + \sum_{t=0}^{T-1} \sum_{i=0}^{R} \max_{p_{t,i}} (\mu_i + p_{t,i}) \mathbb{ED}(p_{t,i})$$

Optimization Methods: Convex optimization, lagrangian relaxation, cutting plane.

Data Generation

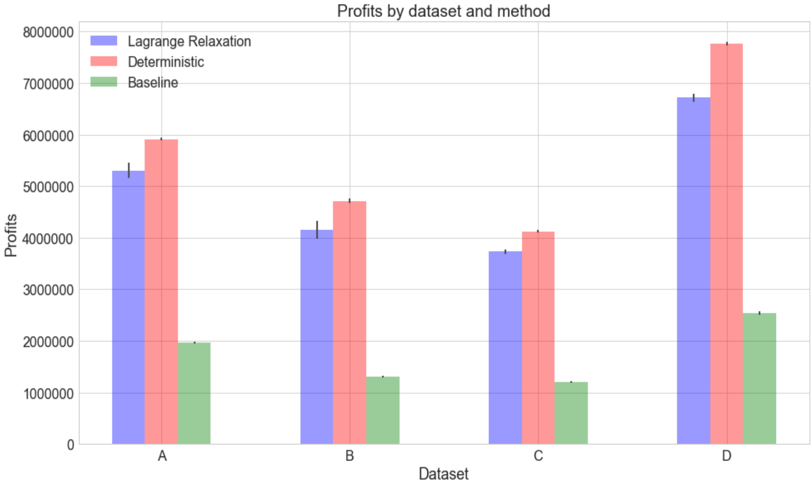
Model data (Input Data):

1. Geography (2 geography types);
2. Airports (5, 10, 15, 20);
3. Planes (5, 10, 15, 20);
4. Circular ways (180 days);
5. Itineraries (50, 100, 500, 1000);
6. Pricing Events (20, 100);
7. Demands $\mathbb{D}(p) \sim Poisson(\lambda e^{-ap})$.

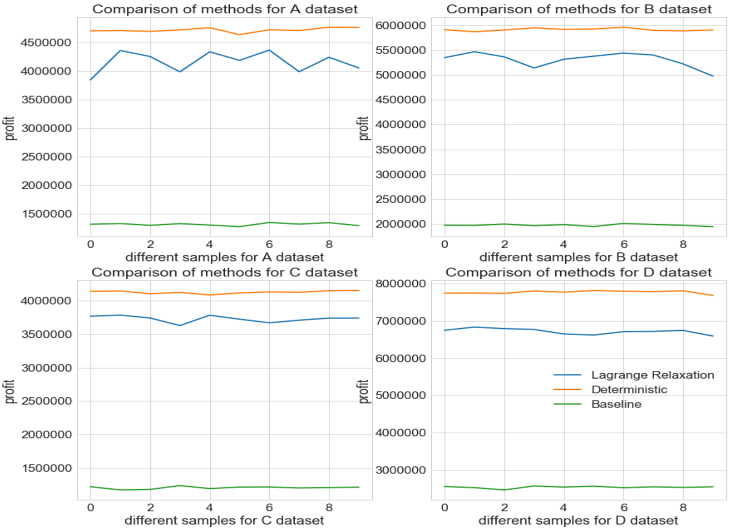
Evaluation data (Market Data):

1. Willingness to pay (Exponential distribution);
2. Show rate modelling (Binomial distribution).

Results


Profits by dataset and method
Lagrange Relaxation
Deterministic
Baseline
8000000
7000000
6000000
5000000
4000000
3000000
2000000
1000000
0
Profits
A
B
C
D
Dataset

Results


Comparison of methods for A dataset
profit
4500000
4000000
3500000
3000000
2500000
2000000
1500000
0
2
4
6
8
different samples for A dataset
Comparison of methods for B dataset
profit
6000000
5500000
5000000
4500000
4000000
3500000
3000000
2500000
2000000
0
2
4
6
8
different samples for B dataset
Comparison of methods for C dataset
profit
4000000
3500000
3000000
2500000
2000000
1500000
0
2
4
6
8
different samples for C dataset
Comparison of methods for D dataset
profit
8000000
7000000
6000000
5000000
4000000
3000000
0
2
4
6
8
different samples for D dataset
Lagrange Relaxation
Deterministic
Baseline

Results

The dependence the score of Lagrange Relaxation method on number of iterations:

# iterations	Time (sec)	Ratio to bound	Bound
200	49.04	0.4773	8840387.088
500	156.46	0.5346	8840387.088
800	337.54	0.5345	8840387.088
1000	559.24	0.535	8840387.088

The ratio to bound for Baseline is 0.1453 and for Deterministic is 0.5333.

Team contribution

We divided the whole project into the following parts:

1. problem proposal and model adaptation - Aliaksandr, Sergey;
2. designing the architecture of the program - Aliaksandr, Aibek;
3. implementing input generation - Aliaksandr, Dmitriy;
4. implementing baseline - Aliaksandr, Sergey;
5. implementing evaluation of the solution - Aliaksandr, Filippos;
6. implementing deterministic approach - Sergey, Aibek;
7. implementing lagrangian relaxation method:
    - deriving an analytic subgradients of the objective function using sympy package - Dmitriy, Aibek;
    - implementing cutting plane method - Filippos, Dmitriy;
    - tuning and acceleration – Aliaksandr, Filippos, Dmitriy, Aibek;
8. writing a report - Aliaksandr, Aibek, Sergey;
9. preparing the final presentation - Aibek, Dmitriy, Filippos.

References

Levin Y., Nediak M., Topaloglu H. — Cargo capacity management with allotments and spot market demand. — // Operations Research. — 2012. — Vol. 60, no. 2. — P. 351–365.

Summary

1. Artificial data modeling;
2. Baseline Approach;
3. Lagrangian Relaxation Approach;
4. Deterministic Apporach;
5. Evaluation;
6. Over 4000 lines of code;
7. Made Filippos speak Russian.

Thank you for your attention! Any questions?