A Mixed Rasch Model of Dual-Process Conditional Reasoning

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Conditional Arguments

- Modus Ponens : {If p then q, p}.$\therefore$ q
- Modus Tollens : {If p then q,$\neg q$}.$\therefore$ $\neg p$
- Affirming Consequent : {If p then q, q}.$\therefore$ p
- Denying Antecedent : {If p then q, $\neg p$}.$\therefore$ $\neg q$

Basic facts

Everybody endorses MP, not very interesting wrt individual differences. We will focus on the 3 remaining arguments. MT is deductively valid, but endorsed by less than 75% of reasoners. Both AC and DA are invalid, yet endorsed by some 50% of reasoners.

The Dual-Process Approach

- System1
  - Pragmatic implicatures
  - Background knowledge
- System2
  - Blocking System1 output
  - Recruiting abstract strategies

2 systems $\times$ 2 mechanisms $=$ 4 subpopulations

Not all System1 responses are the same : Pragmatic implicatures and background knowledge can yield different conclusions. Likewise, not all System2 responses will be the same : Blocking System1 output yields conclusions, but abstract strategies can be recruited to generate others.

Pragmatic System1

- Conversationally, "if p then q" often invites :
  - if $\neg p$ then $\neg q$
  - if q then p

Examples

"If I am promoted, I'll make more money" $\rightsquigarrow$ If he's not, he won't
"If my stocks went down, then I'm broke" $\rightsquigarrow$ If he is, they did

- Invited inferences thus pave the way for AC and DA. But no such conversational route to MT !

The ALL-WRONG pattern

Pragmatic System1 reasoners should endorse AC and DA, but reject MT.
Whatever the *content*, whatever the *argument*, they get it all wrong.

Semantic System1

- Responses to AC, DA, MT are affected by retrieval of information in semantic memory.
  - AC and DA affected by ways to bring up q when p is false
  - MT affected by ways to block q when p is true

Examples

"If he is promoted, he'll be richer " $\rightsquigarrow$ but also if he marries Paris Hilton
"If her stocks went down, then she's broke" $\rightsquigarrow$ unless capital is guaranteed

- Responses in this group should vary according to what type of information comes to mind, depending on the conditional.

No dominant pattern

Semantic System1 reasoners should endorse AC, DA, and MT mainly as a function of *content*. No dominant pattern of answers shall be found, and intermediate endorsement rates are expected for all 3 inferences.

Inhibitory System2

- System2 operates on decontextualized premises
  - inhibition of pragmatic implicatures
  - inhibition of background knowledge

Examples

"If I am promoted, I'll make more money" $\rightsquigarrow$ If he's not, he won't
but must not assume he meant that
"If her stocks went down, then she's broke" $\rightsquigarrow$ unless capital is guaranteed
but must consider that irrelevant

- Inhibition blocks AC and DA, but also the semantic route to MT !

The ALL-BLOCKED pattern

Inhibitory System2 reasoners should reject all three AC and DA, and MT. Correct answers are influenced by *argument* rather than *content*. And they should do worse on MT than semantic System1 reasoners !

Generative System2

- Reasoners now have access to formal strategies
  - premises are still decontextualized
  - reductio ad absurdum is available

Example

"If her stocks went down, then she's broke" $\rightsquigarrow$ unless capital is guaranteed
but must consider that irrelevant
She's not broke. Suppose stocks went down : $\bot$. Hence, they did not.

- Inhibition blocks AC and DA, but a formal route is available to MT

The ALL-CORRECT pattern

Generative System2 reasoners should reject AC and DA, but endorse MT.
Whatever the *content*, whatever the *argument*, they get all the right answers.

Mixed Rasch Model

The mixed Rasch model is an extension of Rasch models and latent class models

One set of parameters for each subpopulation

The mixed Rasch model assumes that within each latent class, a Rasch model holds, but that the values of the parameters of the model can differ between classes

$$P(X_{vi} = 1|g) = \frac{e^{(\theta_{vg} - \sigma_{ig})}}{1 + e^{(\theta_{vg} - \sigma_{ig})}}$$

Probability P of correct response to problem i from individual v in latent class g depends on ability θ_v and the difficulty parameter σ_i of that problem in class g. Item parameters can differ between classes, representing structural differences in the response process.

Predictions

	CONTENT	ARGUMENT	ABILITY	PATTERN
Pragmatic S1	$\sigma \not\propto$	$\sigma \not\propto$	$--$	All-Wrong
Semantic S1	$\sigma \propto$	$\sigma \not\propto$	$-$	—
Inhibitory S2	$\sigma \not\propto$	$\sigma \propto$	$+$	All-Blocked
Generative S2	$\sigma \not\propto$	$\sigma \not\propto$	$++$	All-Correct

Three latent classes

Pragmatic System**1** and generative System**2** are structurally undistinguishable, as problem difficulty should be roughly the same for all problems. Hence, we only expect 3, not 4, latent classes. However, the class with flat distribution of difficulty parameters should split between "all-wrong" and "all-correct" reasoners.

Sample

- 242 men and 244 women, mean age 31 ($SD = 13$)

Education level

20%	Graduate school
41%	College undergraduate
25%	High school graduate
14%	Not a high school graduate

- Lots of students (37%), but the remaining 63% came from practically all professional perspectives (including 10% unemployed).
- Recruitment procedure : Third-year psychology students each asked one man and one woman they knew to take part to the study. Data were collected by the students, usually at the participants' home.

Task

- 6 blocks of 3 syllogisms (AC, DA, MT), each block has different cover story (taken from Thompson, 2000)

The Malaria block

You are a doctor in a tropical country. According to your experience, *if a patient has malaria, he makes a quick recovery*.

MT You observe the following situation : A patient does not make a quick recovery. Does the patient have malaria ? ('Yes', 'No', 'Maybe')

DA You observe the following situation : A patient does not have malaria. Does the patient make a quick recovery ? ('Yes', 'No', 'Maybe')

AC You observe the following situation : A patient makes a quick recovery. Does the patient have malaria ? ('Yes', 'No', 'Maybe')

- Each response was coded '1' when logically correct or else '0'

Best model

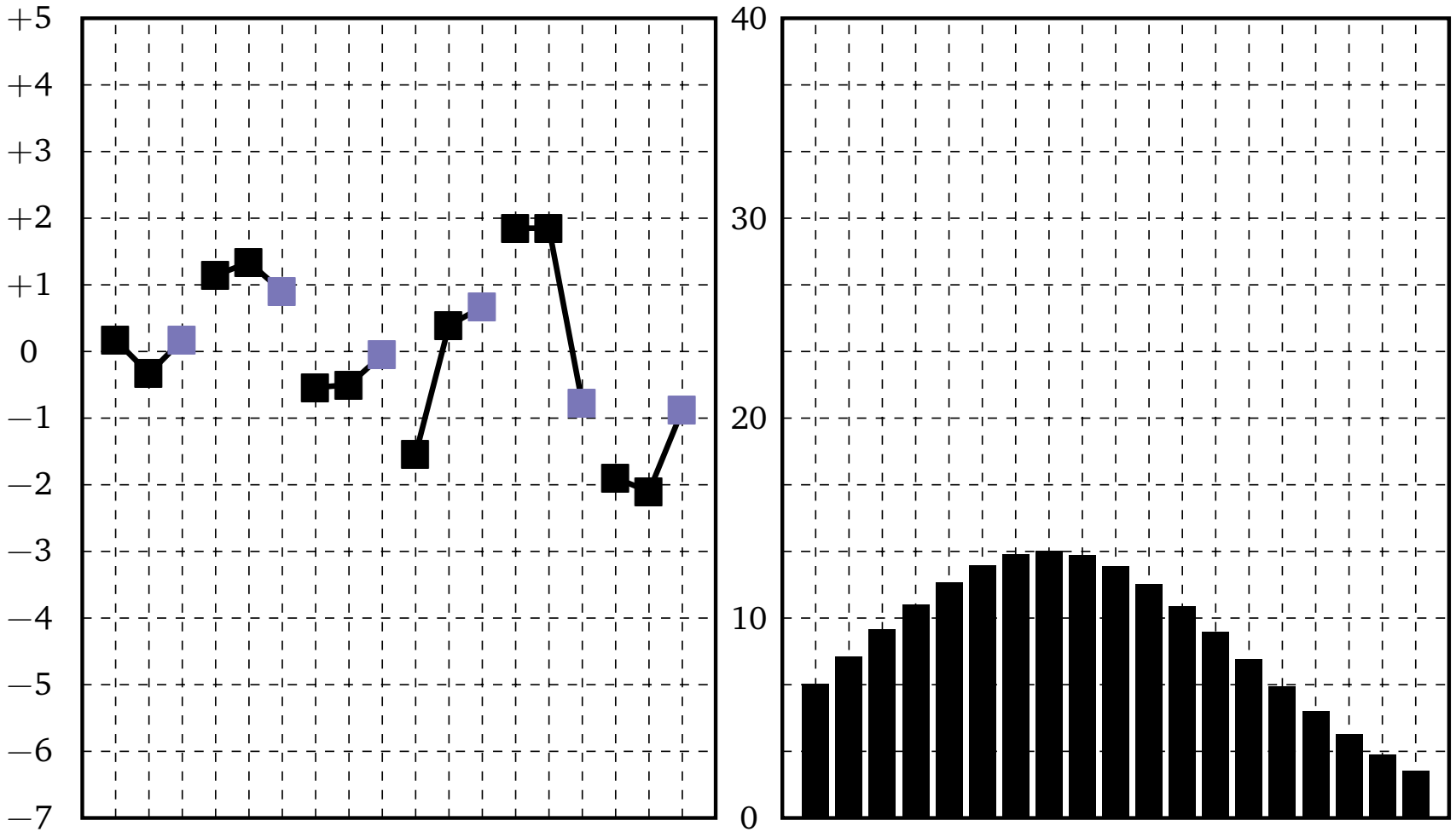
- Mixed Rasch model analysis with WINMIRA
  - Compare 3-class solution to others (Bayesian Information Criterion)
  - Estimate fit with χ^2 and Cressie-Read test (bootstrap method)

CLASSES	BIC	χ^2	CRESSIE-READ
1	8506		
2	8467		
3	8465	$p \simeq .08$	$p \simeq .04$
4	8492		

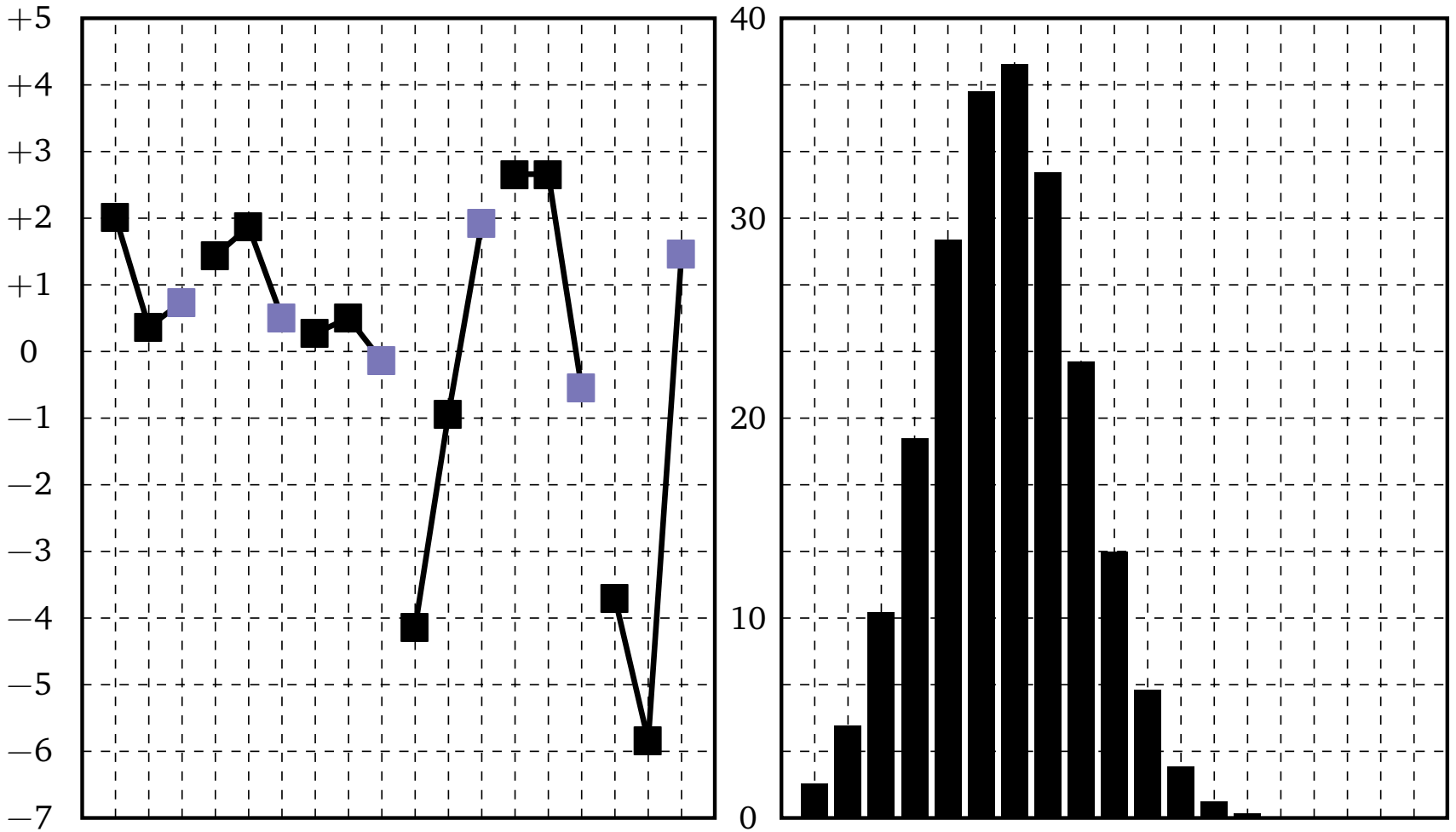
So far, so good

The 3-class solution is preferred to all others, and has acceptable fit.

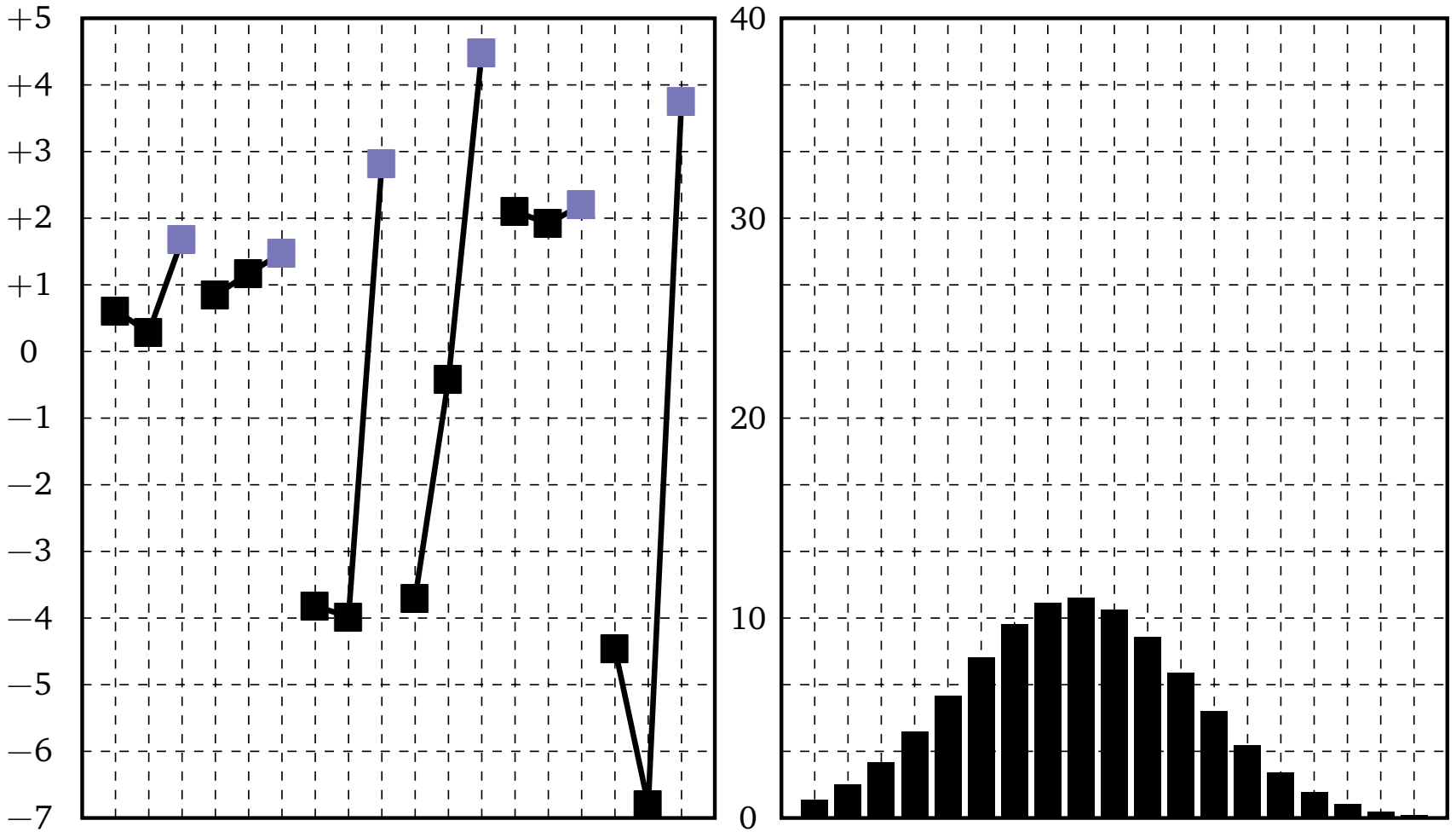
Class A (35%)



Class B (45%)



Class C (20%)



Summary of results

- The 3-class solution has the best fit to the data
- The 3 classes meet the structural expectations

Difficulty of problems

- Class A uninfluenced by content or argument
- Class B influenced primarily by content
- Class C influenced primarily by argument (MT difficult)

- The 3 classes meet the quantitative expectations

Ability of reasoners

- Higher scores in class B than in class C
- Class A has largest proportion of v. low and v. high scores

Perspectives

- Different mechanisms within System1 and System2
- Content/context effects are separable
- Conflict between System1 and System2 outputs is not a prerequisite for System2 to override System1
- Generalization to other tasks ?

The tip of the psychometric iceberg ?

Newstead, Handley, Harley, Wright, & Farelly (2004) : association between general intelligence, resisting AC and DA, and giving normative answer to selection task. Evans, Handley, Neilens, & Over (2006) : association between cognitive sophistication, responses to conditional syllogisms, and responses to truth-table task.