ASSIGNMENT 03 – Power Analysis

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Q1

In the following figure $i_{C_2} = \alpha i_{E_2}$ is the same as $i_{C_2} = \beta_2.i_{B_2}$ where β_2 is the DC current-gain of the Q_2 transistor while V_T is the thermal equivalent voltage. Moreover the relationship between the i_{C_2} and the v_{BE_2} can be written as follows, where I_{s_2} is the saturation current of the Q_2 transistor.

$$i_{C_2} = \alpha i_{E_2} = I_{s_2}.e^{v_{BE_2}/V_T}$$

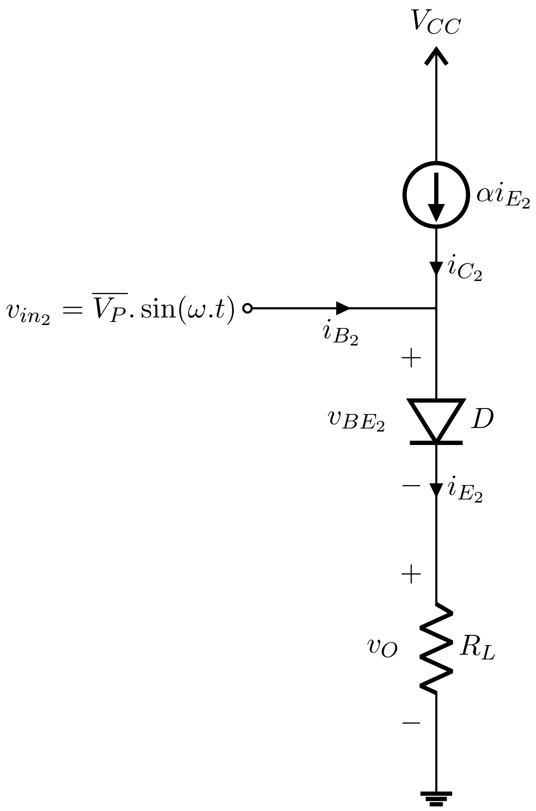

Figure 1: AC Large-Signal Equivalent Circuit for the Output Leg

Q2

Relationship between the v_{in_2} and time(t) is defined as $v_{in_2} = \overline{V_P}.\sin(\omega.t)$, where $\overline{V_P}$ is the peak amplitude and ω is the angular velocity.

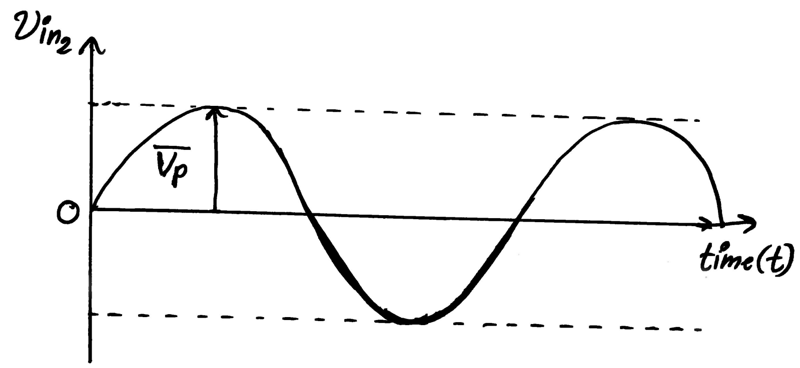

Figure 2: v_{in_2} vs time

Relationship between the $v_L(v_O)$ and time can be derived as follows. Where V_p is the peak amplitude of the ac component of the v_O. Since the voltage gain of stage 2 of the amplifier is $R_L/\left[r_e + R_L\right]$ there will be no phase shift between v_{in_2} and the v_O. Therefore the ac component of the v_O can be directly written as follows.

$$\begin{aligned} v_L = v_O &= i_{E_2}.R_L \\ &= \left(I_{E_2} + i_{e_2}\right).R_L \\ &= I_{E_2}.R_L + i_{e_2}.R_L \\ &= I_{E_2}.R_L + V_p \sin(\omega.t) \\ &= dc\ component + ac\ component \end{aligned}$$

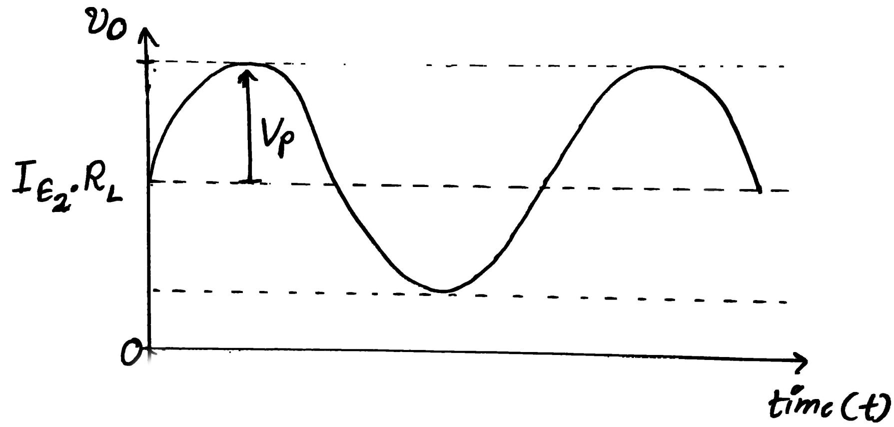

Figure 3: v_L *or* v_O vs time

An expression for the i_{E_2} can be derived as follows. Where I_p is the peak ac current component through R_L. Here again v_O and i_{e_2} will be in-phase sinusoidal signals since resistor is a linear device and i_{e_2} can be expressed as a function of *sine* as follows.

$$\begin{aligned} i_{E_2} &= I_{E_2} + i_{e_2} \\ &= I_{E_2} + \frac{V_p}{R_L}.\sin(\omega.t) \\ &= I_{E_2} + I_p.\sin(\omega.t) \\ &= dc\ component + ac\ component \end{aligned}$$

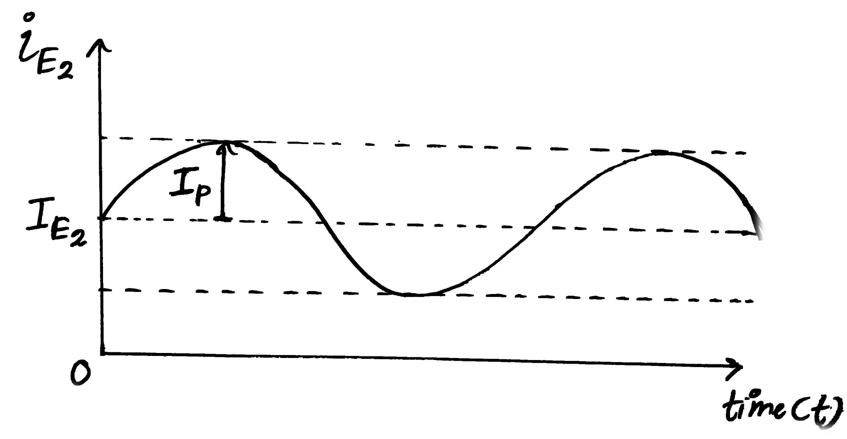

Figure 4: i_{E_2} vs time

Relationship between i_{B_2} and i_{E_2} can be written as follows. $i_{E_2} = (\beta_2 + 1).i_{B_2}$ where β_2 is the DC current-gain of the Q_2 and I_{B_p} is the peak amplitude of the ac component of the i_{B_2}.
Therefore,

$$\begin{aligned}
(\beta_2 + 1).i_{B_2} &= i_{E_2} \\
i_{B_2} &= \frac{1}{\beta_2 + 1}.i_{E_2} \\
&= \frac{1}{\beta_2 + 1}.\left[I_{E_2} + I_p.\sin(\omega.t)\right] \\
&= \frac{1}{\beta_2 + 1}.I_{E_2} + \frac{1}{\beta_2 + 1}.I_p.\sin(\omega.t) \\
&= I_{B_2} + I_{B_p}.\sin(\omega.t) \\
&= \textit{dc component} + \textit{ac component}
\end{aligned}$$

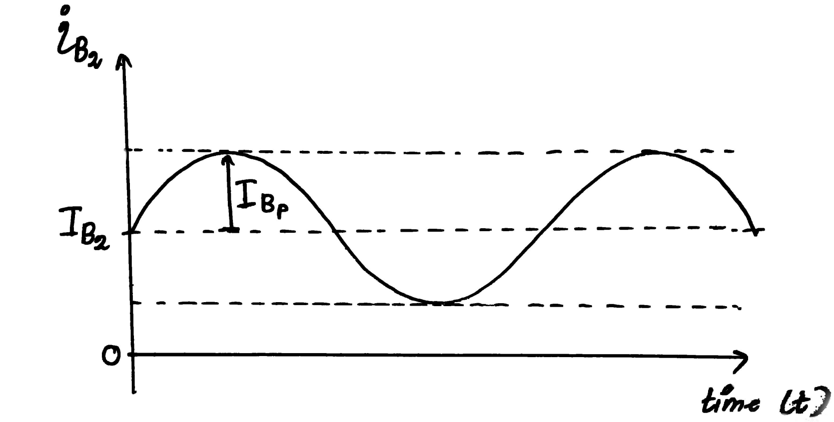

Figure 5: i_{B_2} vs time

By applying Kirchhoff's voltage low to the ***collector-emitter circuit***, $V_{CC} = v_{CE_2} + i_{E_2}.R_L$ and through rearranging this following expression can be obtained.

$$\begin{aligned}
v_{CE_2} &= V_{CC} - i_{E_2}.R_L \\
&= V_{CC} - (I_{E_2} + i_{e2}).R_L \\
&= V_{CC} - I_{E_2}.R_L - i_{e2}.R_L \\
&= (V_{CC} - I_{E_2}.R_L) - V_p\sin(\omega.t) \\
&= \textit{dc component} + \textit{ac component}
\end{aligned}$$

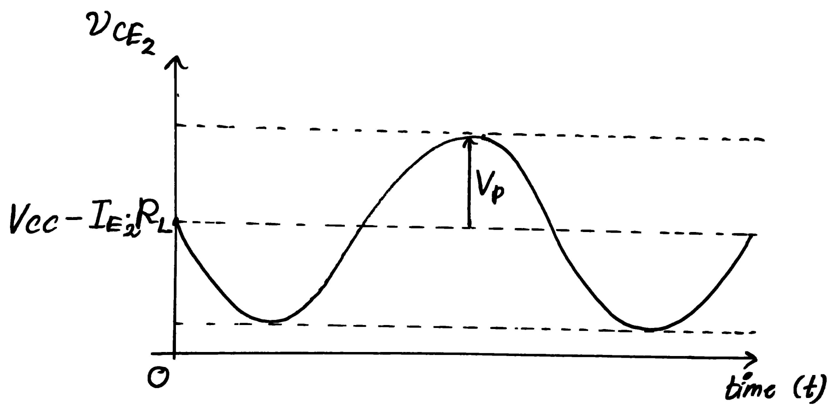

Figure 6: v_{CE_2} vs time

Q3

Derivation of the expressions was done in the previous part.

$$v_{CE_2} = (V_{CC} - I_{E_2}.R_L) - V_p \sin(\omega.t)$$

DC component $= (V_{CC} - I_{E_2}.R_L)$
AC component $= -V_p \sin(\omega.t)$

$$i_{E_2} = I_{E_2} + I_p.\sin(\omega.t)$$

DC component $= I_{E_2}$
AC component $= I_p.\sin(\omega.t)$

Q4

Let the average power delivered to the load R_L be P_L and the period of the *sine* waveform be $T = \frac{2\pi}{\omega}$.

$$\begin{aligned}
P_L &= \frac{1}{T}.\int_0^T (i_{E_2})^2.R_L \ dt \\
&= \frac{1}{T}.\int_0^T (I_{E_2} + I_p.\sin(\omega.t))^2.R_L \ dt \\
&= \frac{1}{T}.\int_0^T \left[(I_{E_2})^2 + (I_p.\sin(\omega.t))^2 + 2.I_{E_2}.I_p.\sin(\omega.t)\right].R_L \ dt \\
&= \frac{1}{T}.\left[\int_0^T (I_{E_2})^2.R_L \ dt + \int_0^T 2.I_{E_2}.I_p.\sin(\omega.t).R_L \ dt + \int_0^T \frac{I_p^2.R_L}{2} \ dt - \int_0^T \frac{I_p^2.R_L.cos(2.\omega.t)}{2} \ dt\right]
\end{aligned}$$

Over a period the average power of a sinusoidal signal is zero. Therefore the second and the fourth terms vanish.

$$\therefore P_L = I_{E_2}^2.R_L + \frac{I_p^2.R_L}{2}$$

Since $V_p = I_p R_L$, the above expression can also be written as follows.

$$\begin{aligned}
P_L &= I_{E_2}^2.R_L + \frac{V_p^2}{2.R_L} \\
&= P_{L,DC} + P_{L,AC}
\end{aligned}$$

Q5

Let the average power delivered to the output leg of the amplifier from DC power supply be P_{DC}. Note that the total current through the output leg is approximated to be i_{E_2}. Therefore,

$$\begin{aligned}
P_{DC} &= \frac{1}{T}.\int_0^T i_{E_2}.V_{CC} \ dt \\
&= \frac{1}{T}.\int_0^T (I_{E_2} + I_p.\sin(\omega.t)).V_{CC} \ dt \\
&= \frac{1}{T}.\left[\int_0^T I_{E_2}.V_{CC} \ dt + \int_0^T I_p.\sin(\omega.t).V_{CC} \ dt\right]
\end{aligned}$$

As mentioned previously over a period the average power of a sinusoidal signal is zero. Therefore the second term vanishes.

$$\therefore P_{DC} = I_{E_2}.V_{CC}$$

Q6

Assume the ***Class A operation*** of the transistor Q_2. Then at the bias point $V_{CE_2} = V_{CC}/2$ and

$$\therefore I_{E_2} = \frac{V_{CE_2}}{R_L} = \frac{V_{CC}/2}{R_L} = \frac{V_{CC}}{2.R_L}$$

$$\begin{aligned}\therefore P_{DC} &= I_{E_2}.V_{CC} \\ &= \frac{V_{CC}}{2.R_L}.V_{CC} \\ &= \frac{V_{CC}^2}{2.R_L}\end{aligned}$$

Let the efficiency of the output leg be $\eta_{output\ leg}$,

$$\begin{aligned}\eta_{output\ leg} &= \frac{AC\ Power\ at\ the\ load(P_{out})}{Input\ Power(P_{in})} \\ &= \frac{P_{L,AC}}{P_{DC}} \\ &= \frac{V_p^2}{2.R_L} \div \frac{V_{CC}^2}{2.R_L} \\ &= \frac{V_p^2}{V_{CC}^2}\end{aligned}$$

Q7

Let overall efficiency of the amplifier be $\eta_{overall}$, total power consumed by the amplifier circuit be P_{total} and total current drained by the amplifier circuit from the source be I_{total}. Then from the Question 6 of Assignment 01- DC Analysis,

$$\begin{aligned}P_{total} &= V_{CC}.I_{total} \\ &= V_{CC}.\left[I_{R_{B_1}} + I_{R_{B_3}} + I_{C_1} + I_{C_2}\right]\end{aligned}$$

Therefore,

$$\begin{aligned}\eta_{overall} &= \frac{AC\ Power\ at\ the\ load(P_{out})}{Total\ Input\ Power(P_{total})} \\ &= \frac{P_{L,AC}}{P_{total}} \\ &= \frac{V_p^2}{2.R_L} \div \left[V_{CC}.\left(I_{R_{B_1}} + I_{R_{B_3}} + I_{C_1} + I_{C_2}\right)\right] \\ &= \frac{V_p^2}{2.R_L.V_{CC}.\left(I_{R_{B_1}} + I_{R_{B_3}} + I_{C_1} + I_{C_2}\right)}\end{aligned}$$