In this project, you’ll write a regular expression matcher similar to `grep`. This has three major steps: first, parse a regular expression into regular operations; second, execute the regular operations to create a NFA; third, run the NFA on input strings. Because we use a linear-time NFA recognition algorithm, our regular expression matcher will actually be much faster than one written using Perl or Python’s regular expression engine. (Most implementations of `grep`, as well as Google RE2, are linear like ours.)

You will need a correct solution for CP1 to complete this project. If your CP1 doesn’t work correctly (or you just weren’t happy with it), you may use the official solution or another team’s solution, as long as you properly cite your source.

Getting started

To make sure your repository is up to date, please have one team member run the commands

```
  git pull https://github.com/ND-CSE-30151-SP20/theory-project-skeleton
  git push
```

and then other team members should run `git pull`. The project repository should then include the following files (among others):

```
bin.{linux,darwin}/
  parse_re
  union_nfa
  concat_nfa
  star_nfa
  string_nfa
  re_to_nfa
  agrep
  compare_nfa
perl/
  agrep.pl
tests/
  test-cp2.sh
```

Please place the programs that you write into the `cp2/` subdirectory.
1 Parser

Note: This part and part 2.1 can be done in parallel.

In this first part, we’ll write a parser for regular expressions. We’ll write the simplest kind of parser, a remeductive parser.

Recursive-descent parsing

To illustrate how to do this, let’s look at the simple grammar from Sipser, Example 2.4. The start symbol is $Expr$.

$$Expr \rightarrow Term \{ + Term \}$$
$$Term \rightarrow Factor \{ * Factor \}$$
$$Factor \rightarrow ( Expr )$$
$$Factor \rightarrow 1$$

We write terminal symbols like this and nonterminal symbols like this. The curly braces mean “zero or more copies of,” exactly like Kleene star. The reason we design the grammar this way (as opposed to Sipser’s Example 2.4) is that it’s more amenable to recursive-descent parsing. The reason we use curly braces (borrowed from Extended Backus-Naur Form) instead of a star is simply to avoid confusion with the Kleene star in regular expressions.

Algorithm 1 shows pseudocode for a recursive-descent parser for this grammar. The function parse takes a string containing an arithmetic expression and returns a tree for the expression. It has a helper function for each nonterminal symbol. The helper function for nonterminal $X$ takes two arguments, $w$ and $i$ ($0 \leq i \leq |w|$), and tries to find a $j$ such that $X \Rightarrow^* w_{i+1} \cdots w_j$. (Note that we’re using 1-based indexing for the symbols of $w$, but $i$ is the number of symbols parsed and so starts at 0.) If there is one, it builds a tree node and returns the node and $j$. If there isn’t such a $j$, it generates an error.

Back to regular expressions

Below is a grammar for regular expressions. The start nonterminal is $Expr$. Let $\Sigma$ be the set of all (ASCII or Unicode) characters.

$$Expr \rightarrow Term \{ | Term \}$$
$$Term \rightarrow \{ Factor \}$$
$$Factor \rightarrow Primary \ast$$
$$Factor \rightarrow Primary$$
$$Primary \rightarrow a \quad a \in \Sigma \setminus \{ (, ), \ast, |, \}$$
$$Primary \rightarrow ( Expr )$$

Implementing the parser for this grammar should be analogous to the one shown for the grammar of arithmetic expressions, except that in the rule for $Term$, there’s no operator between the $Factors$. How do we know when to try to parse another $Factor$ and when to stop? Note that every $Term$ must be followed by the end of the string, $1$, or $\)$. And a $Factor$ can’t start with any of these. So we can look ahead one character, and if it’s the end of the string, $1$, or $\)$, then stop; otherwise, try to parse another $Factor$. 
Algorithm 1: Recursive-descent parser for grammar in Example 2.4.

1: function parse(w)
2:   x, i ← parseExpr(w, 0)
3:   if i = |w| then
4:     return x
5:   else
6:     error
7: function parseExpr(w, i)
8:   x, i ← parseTerm(w, i)
9:   args ← [x]
10:  while w[i+1] = + do
11:     x, i ← parseTerm(w, i+1)
12:     append x to args
13:  return node("add", args), i

14: function parseTerm(w, i)
15:  x, i ← parseFactor(w, i)
16:  args ← [x]
17:  while w[i+1] = * do
18:     x, i ← parseFactor(w, i+1)
19:     append x to args
20:  return node("mul", args), i

21: function parseFactor(w, i)
22:  if w[i+1] = 1 then
23:     return node("const", 1), i+1
24:  else if w[i+1] = ( then
25:     x, i = parseExpr(w, i+1)
26:     if w[i+1] ≠ ) then
27:     error
28:   return x, i+1
29:  else
30:   error
Write a program to test your parser:

\texttt{parse\_re \textit{regexp}}

- \textit{regexp}: a regular expression
- Output: string representation of the syntax tree for \textit{regexp}

The output format should look like a Scheme expression. For example,

\$ \text{parse\_re} \ 'a' \\
(\text{symbol} "a") \\
\$ \text{parse\_re} \ '' \\
(\text{epsilon}) \\
\$ \text{parse\_re} \ '(a)' \\
(group (\text{symbol} "a")) \\
\$ \text{parse\_re} \ 'a*' \\
(\text{star} (\text{symbol} "a")) \\
\$ \text{parse\_re} \ 'abc' \\
(\text{concat} (\text{symbol} "a") (\text{symbol} "b") (\text{symbol} "c")) \\
\$ \text{parse\_re} \ 'a|b|c' \\
(\text{union} (\text{symbol} "a") (\text{symbol} "b") (\text{symbol} "c"))

Note that, to facilitate testing:

- A union or a concat always has two or more arguments. Simplify \((\text{union} \ \alpha)\) and \((\text{concat} \ \alpha)\) into just \(\alpha\), unlike Algorithm 1.
- There should never be a union of unions; flatten \((\text{union} \ (\text{union} \ \alpha \ \beta) \ \gamma)\) or \((\text{union} \ \alpha \ (\text{union} \ \beta \ \gamma))\) into \((\text{union} \ \alpha \ \beta \ \gamma)\). Similarly for concat.

Test your program by running \texttt{test-cp2.sh}.

2 Converter

2.1 Regular operations

Write a function that creates an NFA that accepts exactly one string, and a program to test it:

\texttt{string\_nfa \textit{w}}

- \textit{w}: a string (possibly empty)
- Output: an NFA recognizing the language \{\textit{w}\}

Write functions that perform the three regular operations, using the constructions given in the book, and programs to test them:

\texttt{union\_nfa \textit{M}_1 \ M}_2

- \textit{M}_1, \textit{M}_2: NFAs
- Output: NFA recognizing language \(L(M_1) \cup L(M_2)\)
concat_nfa $M_1$ $M_2$
  • $M_1, M_2$: NFAs
  • Output: NFA recognizing language $L(M_1) \circ L(M_2)$

star_nfa $M$
  • $M$: an NFA
  • Output: NFA recognizing language $L(M)^*$

Test all of these programs by running test-cp2.sh.

2.2 Building the NFA

Write a function that converts (the syntax tree of) a regular expression to an NFA, by walking the tree bottom-up and using the operations implemented in Section 2.1. Then combine your regular expression parser with this function into a test program:

re_to_nfa $regexp$
  • $regexp$: Regular expression
  • Output: NFA $M$ equivalent to $regexp$

Test your program using test-cp2.sh.

3 Putting it together

Finally, combine your regular expression converter with your NFA simulator from CP1 to write a grep replacement, called agrep (for “automaton-based grep”):

agrep $regexp$
  • $regexp$: regular expression
  • Input: strings (one per line)
  • Output: the input strings that match $regexp$

Note that unlike grep, the regular expression should match the entire line, not just part of the line. Test your program by running tests/test-cp2.sh.

The test script also tests the time complexity of agrep. This test is the same as in CP1, but now we can say a bit more about it. For various values of $n$, it creates the regular expression $(a|)(a^n)\text{a}^{2n}$ and tries to match it against the string $\text{a}^{2n}$, using our agrep and yours. For fun, we’ve provided a Perl implementation, called agrep.pl, which you can try for comparison. (I ran out of patience and killed it.)
Submission instructions

Your code should build and run on studentnn.cse.nd.edu. The automatic tester will clone your repository, cd into its root directory, run make -C cp2, and run tests/test-cp2.sh. You’re advised to try all of the above steps and ensure that all tests pass.

To submit your work, please push your repository to Github and then create a new release with tag version cp2 (note that the tag version is not the same thing as the release title). If you are making a partial submission, then use a tag version of the form cp2-123, indicating which parts you’re submitting.

Rubric

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